P1

Monday, March 15, 2021 4:33 PM

– Electromagnetic Waves (Plane Waves) –

We have the following equations from electrostatics:

$$\underbrace{\begin{array}{l} 1\text{-)}\ \bar{\nabla}\times\bar{E} = 0 \\ 2\text{-)}\ \bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0} \end{array}}_{\text{Electrostatic}} \qquad \underbrace{\begin{array}{l} 1\text{-)}\ \bar{\nabla}\times\bar{B} = \mu_0\bar{J} \\ 2\text{-)}\ \bar{\nabla}\cdot\bar{B} = 0 \end{array}}_{\text{Magnetostatic}}$$

Re-arranging these equations. (DC only)

$$\underbrace{\begin{array}{l} 1\text{-)}\ \bar{\nabla}\times\bar{E} = 0 \\ 2\text{-)}\ \bar{\nabla}\times\bar{B} = \mu_0\bar{J} \\ 3\text{-)}\ \bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0} \\ 4\text{-)}\ \bar{\nabla}\cdot\bar{B} = 0 \end{array}}_{\text{Differential forms (point forms)}}$$

Integral forms

1-) $\oint_C \bar{E}\cdot d\bar{l} = 0$ (KVL)

2-) $\oint_C \bar{B}\cdot d\bar{l} = \mu_0 I$ (Ampere's law.)

3-) $\oint_S \bar{E}\cdot d\bar{s} = \frac{Q}{\epsilon_0}$ (Gauss's law)

4-) $\oint_S \bar{B}\cdot d\bar{s} = 0$

Generalized forms: (AC + DC)

1-) $\bar{\nabla}\times\bar{E} = -\frac{d}{dt}\bar{B}$ (Faraday's law) $\longrightarrow \oint_C \bar{E}\cdot d\bar{l} = V_{ind} = \frac{-d}{dt}\phi$

2-) $\bar{\nabla}\times\bar{B} = \mu\left(\bar{J} + \underbrace{\frac{d}{dt}\epsilon\bar{E}}_{\text{Maxwell's contribution.}}\right)$ (Ampere's law) $\longrightarrow \oint_C \bar{B}\cdot d\bar{l} = \mu I + \frac{d}{dt}\mu\oint_S \epsilon\bar{E}\cdot d\bar{s}$

3-) $\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$

4-) $\bar{\nabla}\cdot\bar{B} = 0$

These 4 equations are called "Maxwell's equations", and they are all the equations necessary to solve any electromagnet problem.

—o—

P2

Monday, March 22, 2021 2:25 PM

Constitutive Parameters (Electrical properties of a medium)

1-) Permittivity $= \epsilon$ = gives the amount of polarization.
If ϵ is high, this means highly polarized medium.
For instance, water is such a medium. The E-field is decreased inside such mediums

$$\epsilon = \epsilon_0 \epsilon_r \,, \quad \epsilon_0 = 8.854 \times 10^{-12} \left(\frac{F}{m}\right), \quad \epsilon_r = \text{Relative permittivity}$$

2-) Permeability $= \mu$ = gives the amount of magnetization (e^- spin) of a matter. If μ is high, more magnetization when inserted into a magnetic field. Fe is an example of highly magnetized material (Ferrites).

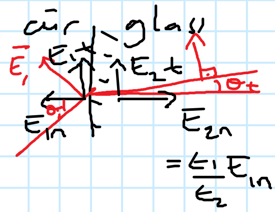
$$\mu = \mu_0 \mu_r \,, \quad \mu_0 = 4\pi \times 10^{-7} \left(\frac{H}{m}\right), \quad \mu_r = \text{relative permeability.}$$

3-) Conductivity $= \sigma$ = gives how much current flows in the presence of E-field in the medium.

$$\bar{J} = \sigma \bar{E} \quad \text{(Ohm's law in points form)}$$

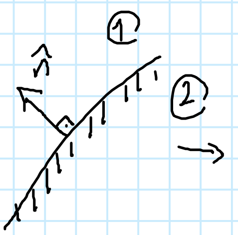
$\bar{J}$: Current density $\left(\frac{A}{m^2}\right)$; $\bar{E}$: Electric field $\left(\frac{V}{m}\right)$

If σ is high like in metals, then small E-field can cause significant current



Boundary Conditions.

Dielectric - Dielectric | Metal - Dielectric | X



	General	Finite conductivity media, no sources or charges $\sigma_1, \sigma_2 \neq \infty$ $\mathcal{J}_s = 0; q_{es} = 0$ $\mathcal{M}_s = 0; q_{ms} = 0$	Medium 1 of infinite electric conductivity $(\mathcal{E}_1 = \mathcal{H}_1 = 0)$ $\sigma_1 = \infty; \sigma_2 \neq \infty$ $\mathcal{M}_s = 0; q_{ms} = 0$	Medium 1 of infinite magnetic conductivity $(\mathcal{E}_1 = \mathcal{H}_1 = 0)$ $\mathcal{J}_s = 0; q_{es} = 0$
Tangential electric field intensity	$-\hat{n} \times (\mathcal{E}_2 - \mathcal{E}_1) = \mathcal{M}_s$	$\hat{n} \times (\mathcal{E}_2 - \mathcal{E}_1) = 0$ → $E_{2t} = E_{1t}$	$\hat{n} \times \mathcal{E}_2 = 0$	$-\hat{n} \times \mathcal{E}_2 = \mathcal{M}_s$
Tangential magnetic field intensity	$\hat{n} \times (\mathcal{H}_2 - \mathcal{H}_1) = \mathcal{J}_s$	$\hat{n} \times (\mathcal{H}_2 - \mathcal{H}_1) = 0$	$\hat{n} \times \mathcal{H}_2 = \mathcal{J}_s$	$\hat{n} \times \mathcal{H}_2 = 0$
Normal electric flux density	$\hat{n} \cdot (\mathcal{D}_2 - \mathcal{D}_1) = q_{es}$	$\hat{n} \cdot (\mathcal{D}_2 - \mathcal{D}_1) = 0$ → $D_{1n} = \epsilon_1 E_{1n}$, $\epsilon_2 E_{2n} = \epsilon_1 E_{1n}$	$\hat{n} \cdot \mathcal{D}_2 = q_{es}$	$\hat{n} \cdot \mathcal{D}_2 = 0$
Normal magnetic flux density	$\hat{n} \cdot (\mathcal{B}_2 - \mathcal{B}_1) = q_{ms}$	$\hat{n} \cdot (\mathcal{B}_2 - \mathcal{B}_1) = 0$	$\hat{n} \cdot \mathcal{B}_2 = 0$	$\hat{n} \cdot \mathcal{B}_2 = q_{ms}$

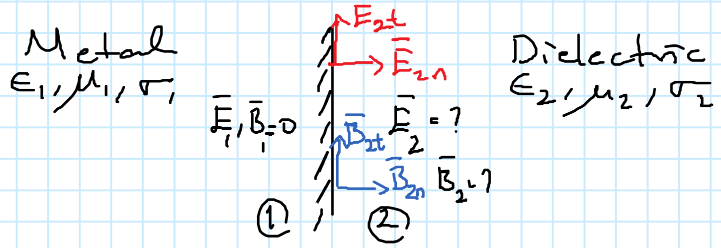
P3

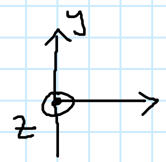
Monday, March 22, 2021 2:36 PM

This table gives us the relation between the fields ($\bar{E}$ and $\bar{B}$) at the boundary of two different mediums.

Ex:

Consider a metal to dielectric interface





Let us suppose that $\bar{E}_1$ and $\bar{B}_1$ at the boundary are known ($\to 0$ for metal).
By using the table, we can obtain $\bar{E}_2$ and $\bar{B}_2$ at the boundary

Find $\bar{E}_2$ and $\bar{B}_2$

Ans

We can use the 2nd column in the table where medium 1 is the metal and medium 2 is the dielectric:

Inside metals: $\bar{E} = 0 \Rightarrow \bar{E}_{1t} = \bar{E}_{1n} = 0$, $B_{1t} = B_{1n} = 0$

$\to$ Tangential $\bar{E}$-field: $\hat{n} \times \bar{E}_{2t} = 0 \Rightarrow -\hat{a}_x \times \bar{E}_{2t} = 0 \Rightarrow \boxed{\bar{E}_{2t} = 0}$

$\to$ Normal $\bar{E}$-field: $\hat{n} \cdot \epsilon_2 \bar{E}_{2n} = \rho_s \left(\frac{C}{m^2}\right) \Rightarrow -\hat{a}_x \cdot \epsilon_2 \bar{E}_{2n} = \rho_s$

$$\Rightarrow -\hat{a}_x \cdot \epsilon_2 \hat{a}_x E_{2n} = \rho_s$$

$$\Rightarrow \boxed{E_{2n} = \frac{\rho_s}{\epsilon_2}} \left(\frac{V}{m}\right)$$

$\to$ Tangential Mag field: $\hat{n} \times \bar{H}_2 = \bar{J}_s \Rightarrow -\hat{a}_x \times \hat{a}_y \frac{B_{2t}}{\mu_2} = -\hat{a}_z J_s$

$$\Rightarrow \boxed{B_{2t} = \mu_2 J_s \left(\frac{Wb}{m^2}\right)}$$

$\to$ Normal mag. field: $\hat{n} \cdot \bar{B}_2 = 0 \Rightarrow \boxed{B_{2n} = 0}$

—o—

P4

Monday, March 22, 2021 3:16 PM

- In summary, $\bar{E}$ and $\bar{B}$ are zero inside metals.
- ρ_s and $\bar{J}_s = 0$ inside dielectrics and $\sigma = 0$
- for metals, $\sigma = \infty$, ρ_s and $J_s \neq 0$

—o—

- Wave Equation -

Let us re-write Maxwell's equations.

Integral forms.

1-) $\bar{\nabla} \times \bar{E} = -\frac{d}{dt}\bar{B}$ (Faraday's law) $\longrightarrow \oint_C \bar{E} \cdot \bar{dl} = V_{ind} = \frac{-d}{dt}\phi$

2-) $\bar{\nabla} \times \bar{B} = \mu \bar{J} + \underbrace{\frac{d}{dt}(\mu \epsilon \bar{E})}_{\text{Maxwell's contribution.}}$ (Ampere's law) $\longrightarrow \oint_C \bar{B} \cdot \bar{dl} = \mu I + \frac{d}{dt}\mu \oint_S \epsilon \bar{E} \cdot \bar{ds}$

3-) $\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon}$ $\longrightarrow \oint_S \bar{E} \cdot ds = \frac{Q}{\epsilon}$

4-) $\bar{\nabla} \cdot \bar{B} = 0$ $\longrightarrow \oint_S \bar{B} \cdot \bar{ds} = 0$

We can solve eqn. (1) and (2) simultaneously to get the wave equation (point forms)

Take the curl of 1^{st} eqn:

$$\bar{\nabla} \times \bar{\nabla} \times \bar{E} = -\bar{\nabla} \times \frac{\partial \bar{B}}{\partial t} = -\frac{\partial}{\partial t}(\bar{\nabla} \times \bar{B})$$

Using the vector identity

$$\bar{\nabla} \times \bar{\nabla} \times \bar{A} = \nabla(\bar{\nabla} \cdot \bar{A}) - \nabla^2 A$$

$\hookrightarrow$ Laplacian operator (2^{nd} order space derivatives)

Then, we get

$$\bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}\underbrace{(\bar{\nabla} \times \bar{B})}$$

$$\bar{\nabla} \times \bar{B} = \mu \bar{J} + \frac{\partial}{\partial t}\mu\epsilon \bar{E} \quad (2^{nd} \text{ eqn.})$$

Then,

$$\bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}\left(\mu \bar{J} + \frac{\partial}{\partial t}\mu\epsilon \bar{E}\right)$$

or

$$\bar{\nabla}\underbrace{(\bar{\nabla} \cdot \bar{E})}_{\frac{\rho}{\epsilon_0}} - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}\mu \bar{J} - \frac{\partial^2}{\partial t^2}\mu\epsilon \bar{E}$$

P5

Monday, March 22, 2021 3:52 PM

$$\bar{\nabla}(\underbrace{\bar{\nabla}\cdot\bar{E}}_{\frac{\rho}{\epsilon}}) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}\mu\bar{J} - \frac{\partial^2}{\partial t^2}\mu\epsilon\bar{E}$$

Also, $\bar{J} = \sigma\bar{E}$

Then, we have

$$\frac{1}{\epsilon}(\bar{\nabla}\cdot\rho) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}(\mu\sigma\bar{E}) - \frac{\partial^2}{\partial t^2}(\mu\epsilon\bar{E})$$

Re-arranging the terms:

$$\boxed{\nabla^2\bar{E} = \frac{1}{\epsilon}(\bar{\nabla}\cdot\rho) + \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}}$$ (Wave equation for $\bar{E}$-field)

We could do the same analysis for $\bar{H}$, we would have

$$\boxed{\nabla^2\bar{H} = \frac{1}{\mu}(\bar{\nabla}\cdot\underbrace{\rho_m}_{0}) + \mu\sigma\frac{\partial\bar{H}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2}}$$ (Wave equ. for $\bar{H}$-field)

$\rho_m \to$ magnetic charge density $= 0$ (fictitious) $\bar{B} = \mu\bar{H}$

Solution of these equations:

Let us use the wave eqn. for $\bar{E}$-field:

$$\nabla^2\bar{E} = \frac{1}{\epsilon}(\bar{\nabla}\cdot\rho) + \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}$$

We want a solution in dielectric medium (air)

Air

ρ $\int$ J $\times$ $\bar{E}, \bar{H}$ $(\bar{E}(x,y,z), \bar{H}(x,y,z)) \longrightarrow z$

In air (dielectric), $\underbrace{\sigma = 0, \rho = J = 0}_{\text{Simple media}}$

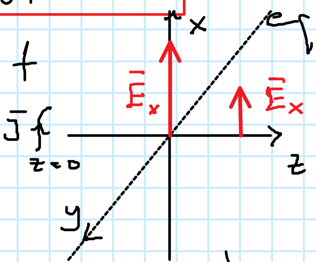
Wave Solution for Simple Medium (Source free and lossless)

$\bar{J} = \rho = 0$ $\sigma = 0$

Then we obtain:

$$\boxed{\nabla^2\bar{E} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} = 0}$$ (Homogenous wave equation for simple medium)

Assume $\boxed{\bar{E} = \hat{a}_x E_x(z,t)}$ type of solution. (plane wave)



$$\Rightarrow \nabla^2\bar{E} = \hat{a}_x\frac{\partial^2 E_x(z,t)}{\partial z^2}$$

Substitute this into the wave equation

P6

Monday, March 22, 2021 4:09 PM

$$\hat{a}_x \frac{\partial^2 E_x(z,t)}{\partial z^2} - \mu\epsilon\, \hat{a}_x \frac{\partial^2 E_x(z,t)}{\partial t^2} = 0$$

Initial conditions = Boundary conditions:

1-) $E_x(0) = E_0$

2-) $E_x\left(\frac{2\pi}{k}\right) = E_0$ (periodic) $k = \omega\sqrt{\mu\epsilon}$, ω = radian frequency.

The solution of this linear 2^{nd} order homogenous differential equation is:

$$\boxed{E_x(z,t) = E_0 \cos(\omega t - kz) \quad \text{(wave solution)}}$$

, where k = wave number

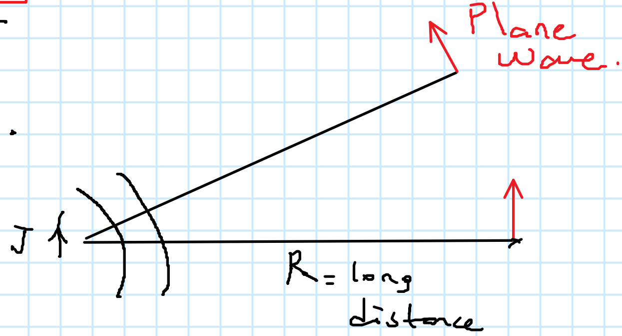
—o—

2 important remarks:

1-) At its origin, $\bar{J}$ causes $\bar{E}$ and $\bar{H}$.

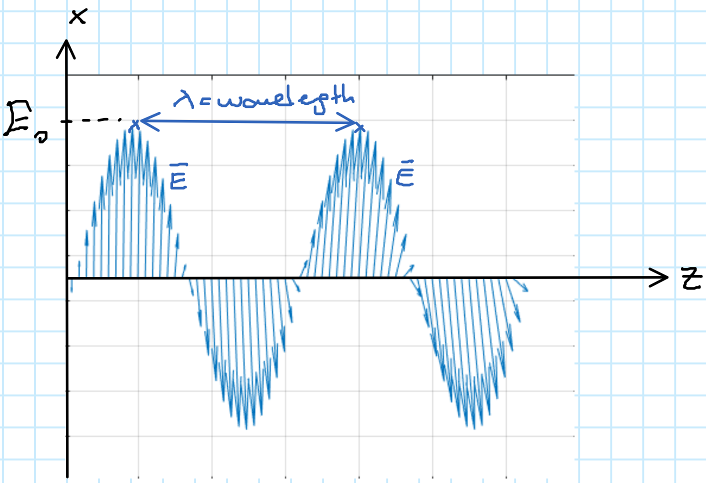
2-) Assumption on the solution

$$\bar{E} = \hat{a}_x E_x(z,t)$$



—o—

Solution:



Phase Velocity:

Consider the plane wave solution

$$E_x(z,t) = E_0 \cos(\omega t - kz) \quad \left(\frac{V}{m}\right)$$

P7

Monday, March 22, 2021 4:49 PM

To find phase velocity

$$\omega t - kz = C_0 = \text{constant}$$

Take the derivative of both sides wrt. t.

$$\Rightarrow \frac{d}{dt}(\omega t) - k\frac{dz}{dt} = \frac{d}{dt} C_0$$

$$\omega - k.v_p = 0$$

$$\Rightarrow \boxed{v_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}}$$

For air, $v_p = c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8$ m/s. (speed of light)

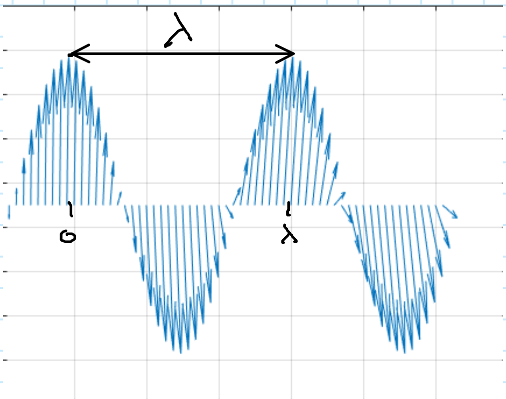
$$v_p\Big|_{\text{water}} = \frac{1}{\sqrt{\mu_0\epsilon_r\epsilon_0}} = \frac{1}{9\sqrt{\mu_0\epsilon_0}} = \frac{3}{9}\times10^8 \text{ m/s}.$$

$\epsilon_r \to 81$

Wavelength:

Consider the plane wave solution

$$E_x(z,t) = E_0\cos(\omega t - kz) \quad \left(\frac{V}{m}\right)$$



At $z=0$: $(t=0)$

$\omega t - kz = 0$

$0 = kz$

$\Rightarrow z = 0$

At $z=\lambda$: $(t=T)$

$\omega t = k\lambda$

$\omega T = k\lambda$

$$\Rightarrow \lambda = \frac{\frac{2\pi}{\cancel{T}}\cdot\cancel{T}}{k}$$

λ = Distance between a cycle of a wave.

$$\boxed{\lambda = \frac{2\pi}{k} \text{ (m)}}$$

or since $k = \omega\sqrt{\mu\epsilon} \Rightarrow \lambda = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{\cancel{2\pi}}{\cancel{2\pi}f\sqrt{\mu\epsilon}} = \frac{1}{f\sqrt{\mu\epsilon}} = \frac{v_p}{f}$

$$\Rightarrow \lambda = \frac{v_p}{f}$$

Ex.
Find the wavelength of a wave at 2.4 GHz (Wi-Fi) in air?

Ans $\lambda = \frac{3\times10^8 \text{ m}/\cancel{s}}{2.4\times10^9 \frac{1}{\cancel{s}}} = \frac{3}{2.4}\times10^{-1} = 0.125$ m $\Rightarrow \lambda = 125$ mm.

P8

Monday, March 29, 2021 3:16 PM

- General plane wave solution has two terms:

$$\boxed{\bar{E}(z,t) = \underbrace{E_0 \cos(\omega t - kz)}_{\text{Incident wave}} + \underbrace{E_0 \cos(\omega t + kz)}_{\text{Reflected wave}}}$$

Wave Solution in Source Free and Lossy Medium:
$\bar{J} = \rho = 0$ $\sigma \neq 0$

Ex: water
concrete.

The general wave equation is given as

$$\nabla^2 \bar{E} = \frac{1}{\epsilon}(\bar{\nabla} \cdot \rho) + \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

For $J = \rho = 0$ (source free) medium, we get

$$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

The solution of this equation is: (Assuming a plane wave)

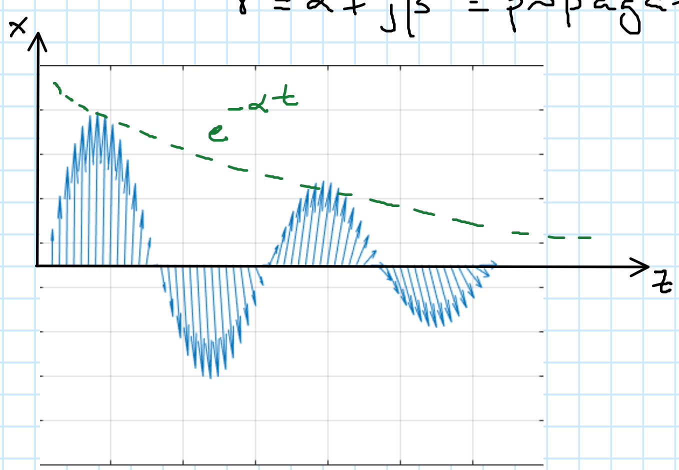
$$\boxed{\bar{E}(z,t) = E_0 e^{-\alpha t} \cos(\omega t - \beta z)}$$

where α = attenuation constant

β = phase constant

and

$\gamma = \alpha + j\beta$ = propagation constant



- For metals, σ is very high, so is α. Thus, wave attenuates very quickly. In fact, only surface charge remains

P9

Monday, March 29, 2021 4:06 PM

- The relationship between α, β and σ and other constants are given in the table below (ω = radian frequency $= 2\pi f$)

E.M. Wave Propagation properties	Exact	Good dielectric $\boxed{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \ll 1}$	Good conductor $\boxed{\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \gg 1}$
Attenuation constant α	$= \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]\right\}^{1/2}$	$\simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
Phase constant β	$= \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]\right\}^{1/2}$	$\simeq \omega\sqrt{\mu\varepsilon}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
Wave Z_w intrinsic η_c impedances $Z_w = \eta_c$	$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$	$\simeq \sqrt{\frac{\mu}{\varepsilon}}$	$\simeq \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$
Wavelength λ	$= \frac{2\pi}{\beta}$	$\simeq \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$	$\simeq 2\pi\sqrt{\frac{2}{\omega\mu\sigma}}$
Velocity v	$= \frac{\omega}{\beta}$	$\simeq \frac{1}{\sqrt{\mu\varepsilon}}$	$\simeq \sqrt{\frac{2\omega}{\mu\sigma}}$
Skin depth δ	$= \frac{1}{\alpha}$	$\simeq \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$	$\simeq \sqrt{\frac{2}{\omega\mu\sigma}}$

Ex: X

For a wave at 2.4 GHz (WiFi/WiMax) find the wave velocity, wavelength, attenuation constant and phase constant for

a-) in air   b-) in sea water   c-) in concrete.

Given that:

$$\epsilon_o = 8.854\times10^{-12}, \quad \mu_o = 4\pi\times10^{-7}, \quad \epsilon_r\Big|_{\text{sea water}} = 70, \quad \epsilon_r\Big|_{\text{concrete}} = 10$$

and

$$\sigma_{air} = 0, \quad \sigma_{\text{sea water}} = 5\ \frac{S}{m}, \quad \sigma_{\text{concrete}} = \frac{1}{100\ \Omega m} = 10^{-2}\ \frac{S}{m}$$

Ans

a-) In air: $\omega = 2\pi f = 2\pi(2.4\times10^{9}) = 1.5\times10^{10} = 15\times10^{9}$ rad/s

We check the condition for $\frac{\sigma}{\omega\epsilon} = 0 \Rightarrow \alpha = 0.$

and $\beta = \omega\sqrt{\mu\epsilon} = 15\times10^{9}\sqrt{\mu_o\epsilon_o} = 15\times10^{9}\sqrt{(4\pi\times10^{-7})(8.854\times10^{-12})} = 50$

Velocity $= \frac{\omega}{\beta} = \frac{15\times10^{9}}{50} = 3\times10^{8}$ m/s, $\quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{50} = \frac{\pi}{25} = 0.125\text{ m} = 12.5\text{ cm}$

P10

Monday, March 29, 2021 4:25 PM

b-) For water: $\dfrac{\sigma}{\omega \epsilon} = \dfrac{5}{(15\times10^9)(70\times 8.854\times10^{-12})} = 0.5378$, $\epsilon \to \epsilon_0 \epsilon_r$

$\frac{1}{10}$th of 1 is considered as " $\ll$ ".

and $\left(\dfrac{\sigma}{\omega\epsilon}\right)^2 = 0.2 \Rightarrow \boxed{0.2 \ll 1}$. Thus, we can use good dielectric approximation.

$$\Rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{5}{2}\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} = \frac{5}{2}\cdot\frac{1}{\sqrt{\epsilon_r}}\underbrace{\sqrt{\frac{\mu_0}{\epsilon_0}}}_{377} = \frac{5\cdot 377}{2}\cdot\frac{1}{\sqrt{70}}$$

$\mu_0 = 4\pi\times10^{-7}$, $\epsilon_0 = 8.854\times10^{-12}$

$$\alpha = 112.65$$

and $\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\cdot\sqrt{\epsilon_r} = 50\cdot\sqrt{70} = 418.$

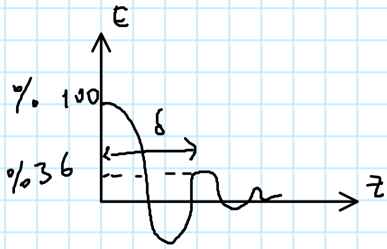
velocity $v = \dfrac{1}{\sqrt{\mu\epsilon}} = \underbrace{\dfrac{1}{\sqrt{\mu_0\epsilon_0}}}_{c}\cdot\dfrac{1}{\sqrt{\epsilon_r}} = \dfrac{c}{\sqrt{\epsilon_r}} = \dfrac{3\times10^8}{\sqrt{70}} = 3.58\times10^7$ m/s. , $\epsilon \to \epsilon_0\epsilon_r$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{418} = 1.5\text{cm}.$$

c-) $\left(\dfrac{\sigma}{\omega\epsilon}\right)^2 = \dfrac{10^{-2}}{(15\times10^9)(70\cdot 8.854\times10^{-12})} = 1.15\times10^{-6} \ll 1$

$\vdots$

HW#4: Complete this example.



Skin Depth:

For lossy mediums, the 'skin depth' is the distance for the electromagnetic wave to lose its amplitude by $\frac{1}{e}$, where $e = 2.71828\ldots$

$\Rightarrow$ Define skin depth as:

$$\boxed{\delta = \frac{1}{\alpha} \ (\text{m.})}$$

The proof:

The plane wave expression of an e.m wave in lossy medium is

$$\bar{E}(z,t) = \hat{a}_x \underbrace{E_0 e^{-\alpha z}}_{E_0} \cos(\omega t - \beta z) \quad \text{(Incident wave)}$$

For $z = \delta = \frac{1}{\alpha}$ $\qquad$ E_0 (max for when $z = 0$).

$$\bar{E}(z=\delta, t) = \hat{a}_x \underbrace{E_0 e^{-\alpha\frac{1}{\alpha}}}_{E_0\cdot\frac{1}{e}} \cos(\omega t - \beta z)$$

P11

Wednesday, July 7, 2021 1:59 PM

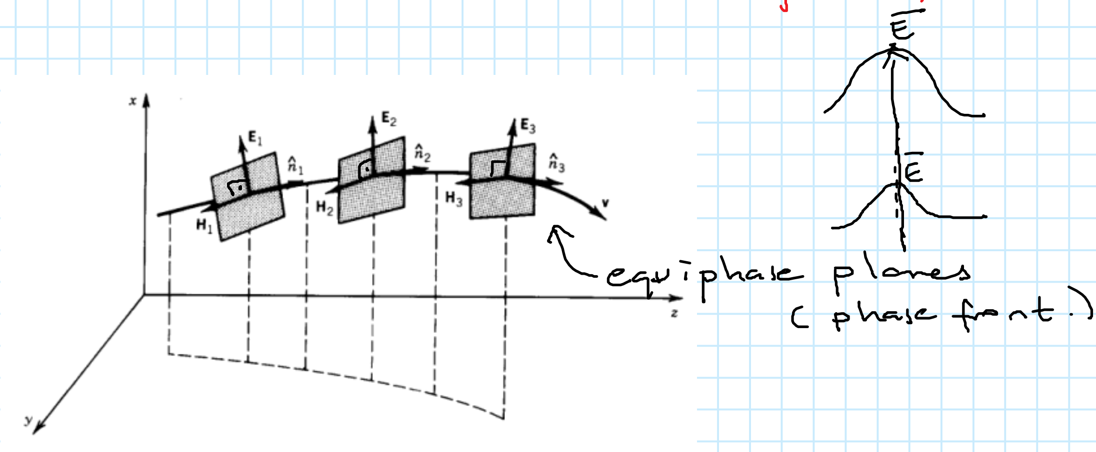
P12

Monday, April 5, 2021 2:40 PM

– Wave Propagation and Plane Waves –

Electromagnetic wave modes are defines as being particular field configurations. They are:

1-) TEM mode = (Transverse Electric and Magnetic)



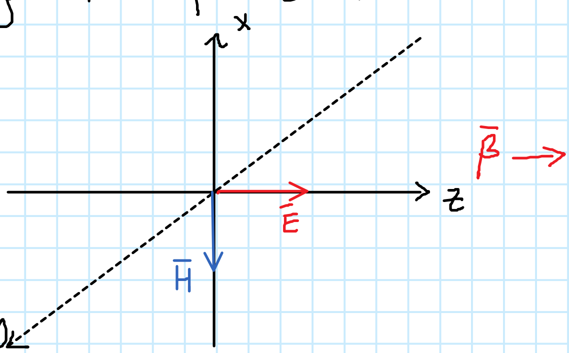
TEM: Transverse Electric and Magnetic.

Electric and Magnetic fields are perpendicular to the direction of propagation.

2-) TM Wave - (Transverse Magnetic)

There is no magnetic field in the direction of propagation.

TM^z: direction of prop.

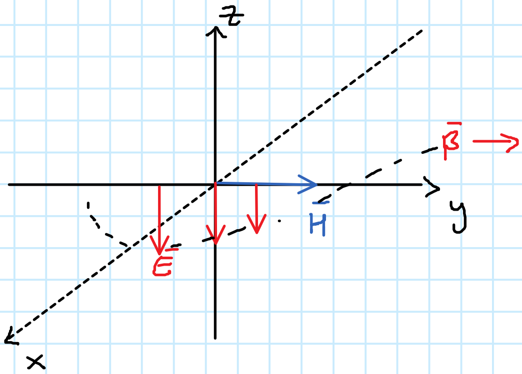


Equiphase plane = xy plane.

3-) TE Wave : (Transverse Electric)

There is no $\bar{E}$-field in the direction of propagation.

TE^y:



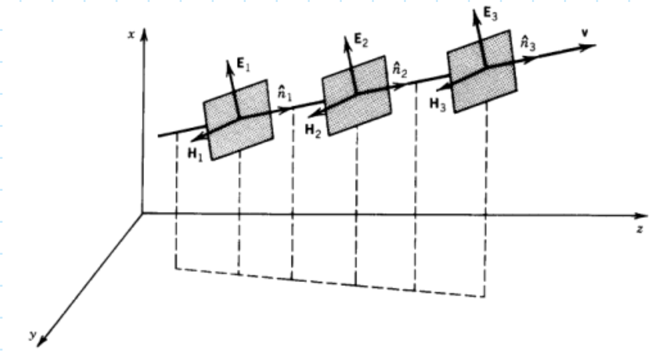
P13

Thursday, July 8, 2021 11:43 AM

Plane Wave:

Electric and magnetic fields are contained in a local plane on which waves have equal phase (equiphase plane).

A plane wave is a TEM wave with planar equiaphase surfaces.



Plane Wave

Uniform Plane Wave: Wave amplitudes are uniform over the equiphase plane.

In TEM mode, no $\bar{E}$ or $\bar{H}$ are on the direction of propagation

$\bar{E}$ and $\bar{H}$ are on the equiphase plane

- All waves converge to a planar at far away from the source.

—o—

- Plane Wave Equations and Time Harmonic Fields -

$\bar{E}(z,t) = \hat{a}_x E_o \cos(\omega t - kz)$, k = wave number

- Since Maxwell's equations and the wave equations are linear, and they are in sinusoidal form, we can use "phasors" to simplify our calculations

$\Rightarrow \bar{E}(z) = \hat{a}_x E_o e^{-jkz}$ (Electric field wave phasor expression.)

$\Downarrow$ Time Harmonic

Similarly for magnetic field:

$\Rightarrow \bar{H}(z) = \hat{a}_y H_o e^{-jkz}$ (for plane wave)

- For plane waves, the general expression in lossless media is:

$$\boxed{\bar{E}(\bar{R}) = \hat{a}_E E_o e^{-j\bar{k}\cdot\bar{R}}}$$

, $\hat{a}_E$ = Unit vector in the direction of $\bar{E}$

where $\bar{R}$ = Position vector from origin to a point on equiphase plane $= \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

$\bar{k}$ = Wavenumber vector $= \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z$

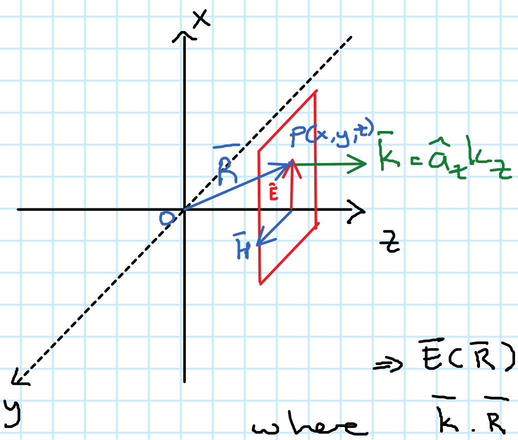
Ex:

Use the general expression for a plane wave electric field which is x directed and moving towards z direction

Ans:

$\bar{k} = \hat{a}_x \underbrace{k_x}_{0} + \hat{a}_y \underbrace{k_y}_{0} + \hat{a}_z k_z$

(Wavenumber vector is in the direction of propagation)



$\bar{R} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

$\Rightarrow \bar{E}(\bar{R}) = \hat{a}_x E_o e^{-j\bar{k}\cdot\bar{R}}$

where $\bar{k}\cdot\bar{R} = (\hat{a}_z k_z)\cdot(\bar{R}) =$

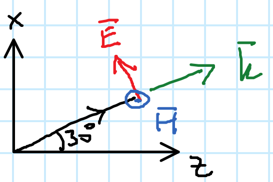
$(\hat{a}_z k_z)\cdot(\hat{a}_x x + \hat{a}_y y + \hat{a}_z z) = k_z z$

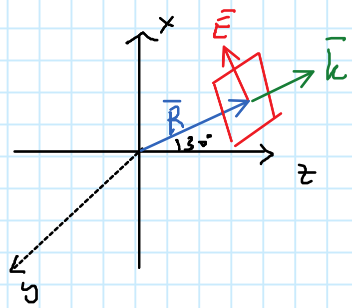
$\Rightarrow \bar{E}(\bar{R}) = \hat{a}_x E_o e^{-jk_z z}$ or

$\bar{E}(z) = \hat{a}_x E_o e^{-jk_z z}$ (Phasor)

Ex. X

Find the electric field expression for a plane wave whose direction of propagation is 30° with the z-axis:





P15

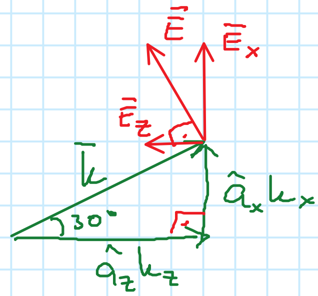
Monday, April 5, 2021 4:02 PM

Ans:

Using

$$\bar{E}(\bar{R}) = \hat{a}_E E_o e^{-j\bar{k}\cdot\bar{R}}$$

where



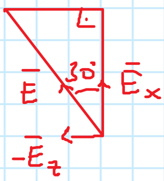
$$\vec{k} = \hat{a}_x k_x + \hat{a}_y \underbrace{\cancel{k_y}}_{0} + \hat{a}_z k_z = \hat{a}_x k_x + \hat{a}_z k_z$$

$$= \hat{a}_x k \sin 30 + \hat{a}_z k \cos 30°$$

$$= \hat{a}_x \frac{k}{2} + \hat{a}_z \frac{\sqrt{3}\,k}{2}$$

$$\bar{R} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z.$$

$$\Rightarrow \bar{k}\cdot\bar{R} = x\frac{k}{2} + z\frac{k\sqrt{3}}{2}$$



Also, $\bar{E} = \hat{a}_x \underbrace{E_x}_{\bar{E}_x \,\to\, E_o\cdot\cos 30} - \hat{a}_z \underbrace{E_z}_{\bar{E}_z \,\to\, E_o\cdot\sin 30}$

$$\Rightarrow \boxed{\bar{E}(\bar{R}) = E_o\left(\hat{a}_x\frac{\sqrt{3}}{2} - \frac{\hat{a}_z}{2}\right)e^{-jk\left(\frac{x}{2}+\frac{z\sqrt{3}}{2}\right)} \quad \left(\frac{V}{m}\right)} \text{ (Phasor)}$$

—o—

- For a plane wave:

$$\boxed{\bar{H}(\bar{R}) = \frac{1}{\eta}\,\hat{a}_n \times \bar{E}(\bar{R}) \quad \left(\frac{A}{m}\right)}$$

where $\hat{a}_n$ = Unit vector in the direction of propagation.

and η = Point impedance or intrinsic impedance.

and $\eta = Z_w = \sqrt{\frac{\mu}{\varepsilon}}$ (for dielectrics)

For air $\eta = \sqrt{\frac{\mu_o}{\varepsilon_o}} = \sqrt{\frac{4\pi\times10^{-7}}{8.854\times10^{-12}}} = 377\,\Omega.$

For example, in the previous question if we wanted to find $\bar{H}$:

$$\hat{a}_n = \frac{\bar{k}}{\|\bar{k}\|} = \frac{\hat{a}_x\frac{k}{2} + \hat{a}_z\frac{\sqrt{3}\,k}{2}}{\sqrt{\frac{k^2}{4}+\frac{3}{4}k^2}} \qquad , \; k = \omega\sqrt{\mu\varepsilon} = \frac{\omega}{c}$$

⋮

P16

Monday, April 5, 2021 4:40 PM

Wave Power:

Instantaneous wave power density: $\boxed{\bar{P} = \bar{E} \times \bar{H} \ \left(\frac{W}{m^2}\right)}$

(Poynting vector)

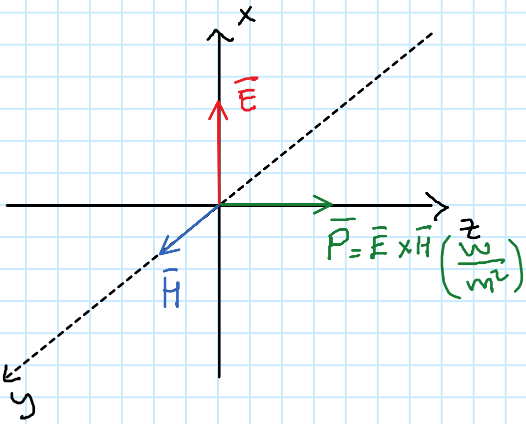
Total Wave power,

$$\boxed{P_{total} = \int_{sphere} \bar{P} \cdot \bar{ds}} \ (W)$$

Average Power Density:

$$\boxed{\bar{P}_{avg} = \frac{1}{2} (\bar{E} \times \bar{H}^*)} \quad (W/m^2)$$

$$\boxed{\bar{P}_{avg} = \bar{E}_{eff} \times \bar{H}^*_{eff}} \quad (W/m^2)$$



—o—

Ex: X

Find the electric and magnetic field plane waves for ($\eta = 377\Omega$). Also, find $\bar{P}$ and $\bar{P}_{avg}$.

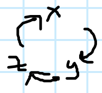
$$\bar{E} = \hat{a}_x E_o e^{-jkz} \quad \text{(incident wave)}.$$

Ans.

$\bar{k} = \hat{a}_z k_z$ (propagation is in the z-direction.)

$\hat{a}_n = \hat{a}_z$

$$\Rightarrow \quad \bar{H}(\bar{R}) = \frac{1}{\eta} \hat{a}_n \times \bar{E}(\bar{R})$$



$$= \frac{1}{377} \hat{a}_z \times \hat{a}_x E_o e^{-jkz}$$

$$= \frac{1}{377} \hat{a}_y E_o e^{-jkz} = \boxed{\hat{a}_y \frac{E_o}{377} e^{-jkz}}$$

The instantaneous power density vector:

P17

Monday, April 12, 2021 2:33 PM

$$\bar{P} = \bar{E}\times\bar{H} = \hat{a}_x E_o e^{-jkz} \times \hat{a}_y \frac{E_o}{\eta} e^{-jkz} = \boxed{\hat{a}_z \frac{E_o^2}{\eta} e^{-2jkz} \quad \left(\frac{w}{m^2}\right)}$$

Note that

$$Re[(\bar{E}\times\bar{H})e^{j\omega t}] \neq Re[\bar{E}.e^{j\omega t}] \times Re[\bar{H}\, e^{j\omega t}]$$

Because the evaluation of power is non-linear.
(E is multiplied by H).

$$\Rightarrow P(z,t) = Re[\bar{E}.e^{j\omega t}] \times Re[\bar{H}.e^{j\omega t}]$$

$$= \hat{a}_x E_o \cos(\omega t - kz) \times \hat{a}_y \frac{E_o}{\eta} \cos(\omega t - kz)$$

$$= \hat{a}_z \frac{E_o^2}{\eta} \cos^2(\omega t - kz)$$

$$= \hat{a}_z \frac{E_o^2}{\eta} \cos(2\omega t - 2kz) \quad \left(\frac{w}{m^2}\right)$$ (Time dependent expression)

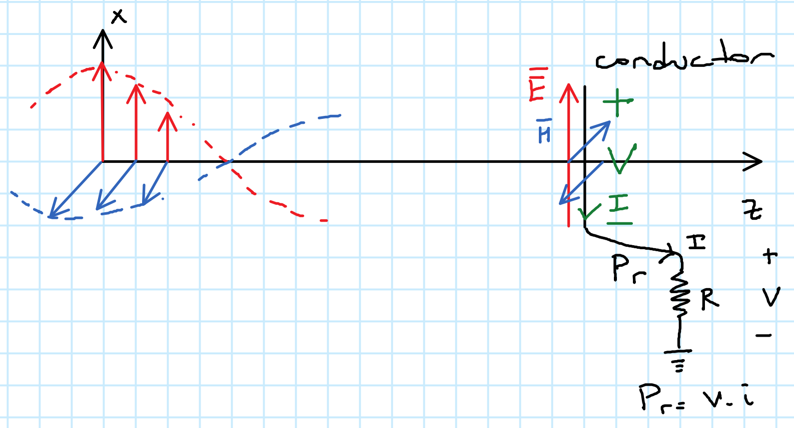
$$P_{avg} = \frac{1}{2} Re[\bar{E}\times\bar{H}^*] = \boxed{\frac{E_o^2}{2\eta}} \quad \left(\frac{w}{m^2}\right)$$

In case of a lossy media-

$$P(z,t) = \hat{a}_z \frac{E_o^2}{\eta} e^{-2\alpha z} \cos(2\omega t - 2kz) \quad \left(\frac{w}{m^2}\right)$$

—o—

- There is a relation between fields and circuit quantities:

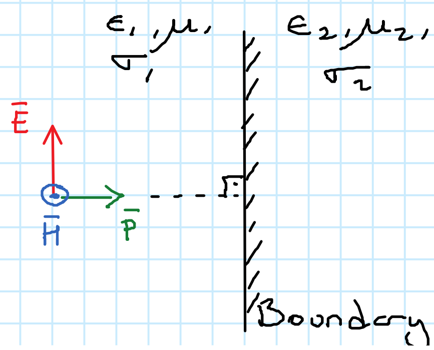


$\boxed{V = E.d}$

$\boxed{2\pi r H = I}$

- Reflection and Transmission of Waves -

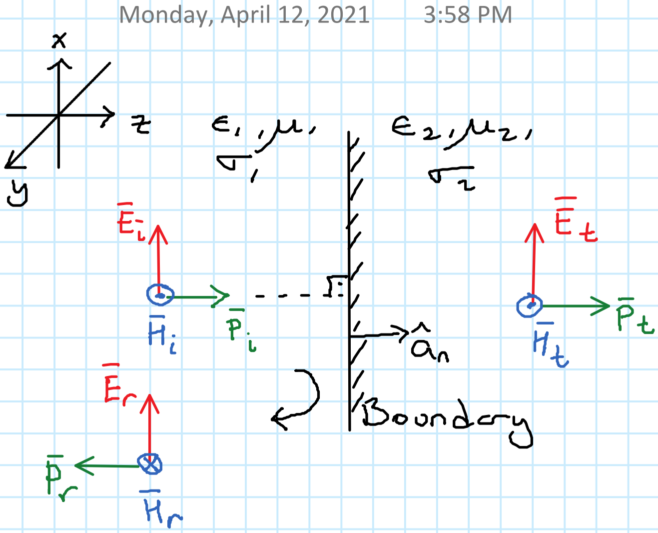
1-) Normal Interface:
(Lossless case)



Boundary

P18

Monday, April 12, 2021 3:58 PM



$P_r + P_t = P_i$ (conservation of power)

Analysis:

$$\bar{E}_i = \hat{a}_x E_o e^{-jk_1 z}$$, incident wave in lossless medium.

$$\bar{E}_r = \hat{a}_x \underbrace{\Gamma E_o}_{E_r} e^{+jk_1 z}$$, reflected wave where k_1 is the wavenumber of the 1st medium.

where

$$\boxed{\Gamma = \text{Reflection coefficient} = \frac{E_r}{E_i}}$$

If $\Gamma = 0 \Rightarrow E_r = 0$ (Total transmission)

If $\Gamma = 1 \Rightarrow E_r = E_i$ (Total reflection)

$$\Rightarrow |\Gamma| = \frac{|E_r|}{E_o} \quad , \quad 0 \leq |\Gamma| \leq 1$$

Similarly,

$$\bar{E}_t = \hat{a}_x \underbrace{T E_o}_{E_t} e^{-jk_2 z}$$, transmitted wave in the 2nd medium.

$$\Rightarrow \boxed{T = \text{Transmission coefficient} = \frac{E_t}{E_i}} \quad , \quad 0 \leq |T| \leq 1$$

Also,

$$\bar{H}_i = \hat{a}_y \frac{E_o}{\eta_1} e^{-jk_1 z}$$

$$\bar{H}_r = -\hat{a}_y \Gamma \frac{E_o}{\eta_1} e^{+jk_1 z}$$

$$\bar{H}_t = \hat{a}_y T \frac{E_o}{\eta_2} e^{-jk_2 z}$$

P19

Monday, April 12, 2021 4:21 PM

- Imposing the B.C's:

$$|E_1^t| = |E_2^t| \Rightarrow |\bar{E}_i| + |\bar{E}_r| = |\bar{E}_t|$$

$$E_o + \Gamma E_o = T E_o$$

$$\boxed{1 + \Gamma = T} \text{ (1)}$$

- Imposing another B.C's:

$$|\bar{H}_1^t| = |\bar{H}_2^t| \Rightarrow \frac{E_o}{\eta_1} - \frac{\Gamma E_o}{\eta_1} = \frac{T E_o}{\eta_2}$$

$$\Rightarrow \boxed{\frac{1}{\eta_1}(1-\Gamma) = \frac{1}{\eta_2} T} \text{ (2)}$$

Solving (1) and (2) simultaneously,

$$\Rightarrow \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \text{ and } \boxed{T = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

- Thus, we can obtain Γ and T, if we know the wave impedances of the mediums

Power Considerations:

$$\bar{P}_{i_{avg}} = \frac{1}{2} Re(\bar{E}_i \times \bar{H}_i^*) = \boxed{\hat{a}_z \frac{E_o^2}{2\eta_1}} \left(\frac{W}{m^2}\right)$$

$$\bar{P}_{r_{avg}} = \frac{1}{2} Re[\bar{E}_r \times \bar{H}_r^*] = \boxed{-\hat{a}_z |\Gamma|^2 |P_{i_{avg}}| \quad \left(\frac{W}{m^2}\right)}$$

$$\bar{P}_{t_{avg}} = \frac{1}{2} Re[\bar{E}_t \times \bar{H}_t^*] = \hat{a}_z |T|^2 \frac{\eta_1}{\eta_2} \cdot |\bar{P}_{i_{avg}}|$$

$$= \boxed{\hat{a}_z [1 - |\Gamma|^2] |\bar{P}_{i_{avg}}|}$$

P20

Monday, April 12, 2021 4:43 PM

Ex:

A uniform plane wave in air is incident normally upon a flat lossless medium with $\epsilon_r = 2.56$ (polystrene). Determine the reflection and transmission coefficients and the power densities in each medium. Assume that the amplitude of the incident E-field is 1mV/m

Ans:

$$E_o = |\bar{E}_i| = 1 \frac{mV}{m}.$$

Wave impedances. $Z = \sqrt{\frac{\mu}{\epsilon}} = \eta.$

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_o}{\epsilon_o}} \quad \mu_o \leftarrow 4\pi \times 10^{-7}, \quad \epsilon_o \leftarrow 8.854\times 10^{-12}.$$

and

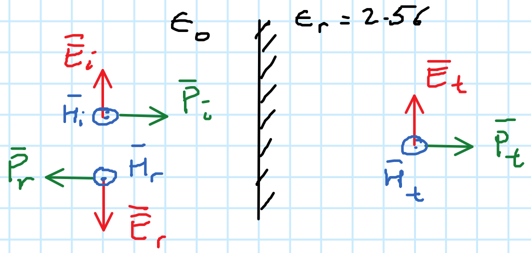
$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_o}{2.56\,\epsilon_o}} = \frac{1}{1.6}\sqrt{\frac{\mu_o}{\epsilon_o}} = \frac{1}{1.6}\eta_1$$

Thus, Reflection & Transmission coefficients are

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{1.6}\eta_1 - \eta_1}{\frac{1}{1.6}\eta_1 + \eta_1} = \frac{\frac{1}{1.6} - 1}{\frac{1}{1.6} + 1} = \frac{1 - 1.6}{1 + 1.6} = -0.231$$

$$\Rightarrow T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\left(\frac{1}{1.6}\right)}{1 + \frac{1}{1.6}} = \frac{2}{2.6} = 0.769$$

$$\Rightarrow 1 + \Gamma = T \quad \Rightarrow 1 - 0.231 = 0.769 \checkmark \text{ from eqn (1)}$$



Power Calculations.

$$|P^i_{avg}| = \frac{E_o^2}{2\eta_1} = \frac{(10^{-3})^2}{2(377)} = 1.327\times 10^{-9} = 1.327 \frac{nW}{m^2}$$

$$|P^r_{avg}| = |\Gamma|^2 |\bar{P}^i_{avg}| = |-0.231|^2 (1.327\times 10^{-9}) = 0.071 \frac{nW}{m^2}$$

$$|P^t_{avg}| = (1 - |\Gamma|^2)|\bar{P}^i_{avg}| = |T|^2 \frac{\eta_1}{\eta_2} |P^i_{avg}| = [1 - (0.231)^2]\cdot(1.327\times 10^{-9})$$

$$\Rightarrow |P^i_{avg}| = |P^r_{avg}| + |P^t_{avg}| \qquad = 1.256 \frac{nW}{m^2}$$

P21

Monday, April 19, 2021 2:30 PM

Ex

Repeat the previous example for air to water interface.

$\epsilon_r\Big|_{water} = 81$, $\mu_r\Big|_{water} = 1 \Rightarrow \mu_{water} = \mu_0$.

Ans

$E_0 = |\bar{E}_i| = 1 \frac{mV}{m}$.

Wave impedances. $Z = \sqrt{\frac{\mu}{\epsilon}} = \eta$.

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \begin{matrix}\leftarrow 4\pi \times 10^{-7} \\ \leftarrow 8.854 \times 10^{-12}\end{matrix} = 377\Omega. \text{ (wave impedance of air)}$$

and

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_0}{81\epsilon_0}} = \frac{1}{9}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{9}\eta_1$$

Thus, Reflection & Transmission coefficients are

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{9}\eta_1 - \eta_1}{\frac{1}{9}\eta_1 + \eta_1} = \frac{\frac{1}{9} - 1}{\frac{1}{9} + 1} = \frac{1-9}{1+9} = -\frac{8}{10} = -0.8$$

$$\Rightarrow T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\left(\frac{1}{9}\right)}{1 + \frac{1}{9}} = 0.2$$

Thus, $|\bar{E}_r| = |\bar{E}_i||\Gamma| = (1mV)(0.8) = 0.8\,mV$

$|\bar{E}_t| = |\bar{E}_i||T| = 0.2\,mV$

$$|P^i_{avg}| = \frac{E_0^2}{2\eta_1} = \frac{(10^{-3})^2}{2(377)} = 1.327 \times 10^{-9} = 1.327 \frac{nW}{m^2}$$

$$|P^r_{avg}| = |\Gamma|^2 |\bar{P}^i_{avg}| = (-0.8)^2 (1.327 \times 10^{-9}) = 8.5 \times 10^{-10} = 0.85 \frac{nW}{m^2}$$

$$|P^t_{avg}| = (1 - |\Gamma|^2)|\bar{P}^i_{avg}| = |T|^2 \frac{\eta_1}{\eta_2} |P^i_{avg}| = [1 - (0.8)^2] \cdot (1.327 \times 10^{-9}) = 0.47 \frac{nW}{m^2}$$

$\Rightarrow |P^i_{avg}| = |P^r_{avg}| + |P^t_{avg}|$ ✓

P22

Monday, April 19, 2021 2:43 PM

Ex:

For the same electric field plane wave with $E_0 = 1\,\frac{mV}{m}$ consider air to aluminum interface. Find Γ and $T = ?$

For Al $\rightarrow \sigma = 3.54\times10^{7}\,\frac{S}{m}$, $\epsilon_r = 10$. $f = 1\,GHz$ (GSM phone comm.)

Ans:

$\eta_1 = 377\,\Omega$ (air), $\eta_2 = ?$ $\Rightarrow \left(\frac{\sigma}{\omega\epsilon}\right)^2 = \left(\frac{3.54\times10^{7}}{2\pi\times10^{9}\times10\times8.854\times10^{-12}}\right)^2 \gg 1$

$$\Rightarrow \eta_2 = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = \sqrt{\frac{2\pi\times10^{9}\times4\pi\times10^{-7}}{2(3.54\times10^{7})}}(1+j)$$

$$= 0.015\,e^{j0.7854}$$

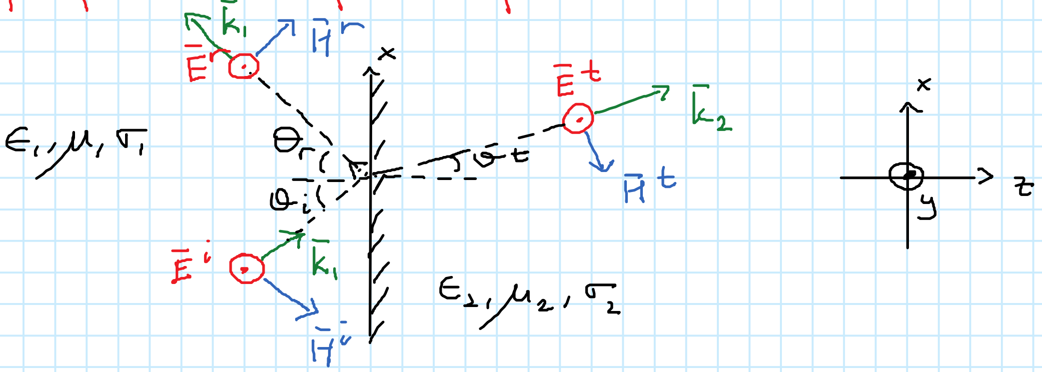
$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{0.015e^{j0.78} - 377}{0.015e^{j0.78} + 377} = 0.9999\,e^{j3.14\,rad} = 0.9999\,e^{j180^\circ}$$

$$\bar{E}_r = \bar{E}_i\cdot\Gamma = \hat{a}_x E_0 e^{-jkz}\cdot 0.9999\,\underbrace{e^{j180^\circ}}_{-1}$$
$$= -\hat{a}_x E_0 e^{jkz}$$

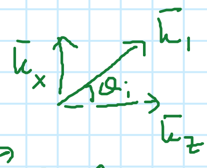
z —o—

2-) Angled Incidence (Oblique Incident) (Lossless Media).

a-) Perpendicular (TE mode) Polarization: shows the directional properties of the fields.



where θ_i = Angle of incidence
θ_r = Angle of reflection
θ_t = Angle of transmission



and

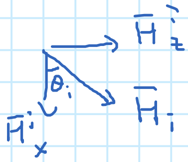
$$\bar{E}_i = \hat{a}_y E_0\, e^{-j\,\underbrace{k_1(x\sin\theta_i + z\cos\theta_i)}}$$

$\vec{k} = \hat{a}_x k_x + \hat{a}_y k_y$

$\bar{k}\cdot\bar{R}$, $\hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

P23

Monday, April 26, 2021 2:33 PM



$$H_x^i = \hat{a}_x \frac{E_0}{\eta} \cos\theta_i$$
$$\bar{H}_z^i = \hat{a}_z \frac{E_0}{\eta} \sin\theta_i$$

$$\bar{H}^i = (-\hat{a}_x \cos\theta_i + \hat{a}_z \sin\theta_i)\frac{E_0}{\eta_1} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

where $\bar{k}_x = \hat{a}_x k_1 \sin\theta_i$, $\bar{k}_z = \hat{a}_z k_1 \cos\theta_i$

$$\bar{E}^r = \hat{a}_y \Gamma_{TE} E_0 e^{-jk_1(x\sin\theta_i - z\cos\theta_i)}$$
$$\bar{H}^r = (\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r)\,\Gamma_{TE}\frac{E_0}{\eta_1} e^{-jk_1(x\sin\theta_i - z\cos\theta_i)}$$

Similarly,

$$\bar{E}^t = \hat{a}_y T_{TE} E_0 e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$
$$\bar{H}^t = (-\hat{a}_x \cos\theta_t + \hat{a}_z \sin\theta_t)\cdot T_{TE}\cdot\frac{E_0}{\eta_2} e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

Boundary Conditions:

1-) $\bar{E}^i + \bar{E}^r \Big|_{\substack{tan \\ z=0}} = \bar{E}^t \Big|_{\substack{tan \\ z=0}}$

2-) $\bar{H}^i + \bar{H}^r \Big|_{\substack{tan \\ z=0}} = \bar{H}^t \Big|_{\substack{tan \\ z=0}}$

There are 2 B.C.'s that give us 4 equations when we equate the real and imaginary parts among themselves. These 4 equations give us the relation among θ_r, θ_t, Γ_{TE} and T_{TE}

The following relations are obtained:

1-) $\boxed{\theta_r = \theta_i}$ (Snell's law of reflection)

2-) $\boxed{k_1 \sin\theta_i = k_2 \sin\theta_t}$ (Snell's law of refraction or transmission)

k_1 and k_2 can be obtained from the Table in pg. 9

For lossless dielectrics , $k_1 = \omega\sqrt{\mu_1\varepsilon_1}$, $k_2 = \omega\sqrt{\mu_2\varepsilon_2}$ ($\bar{k}$ = wave number)

Also,

$$\boxed{\Gamma_{TE} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

$T_{TE} = \frac{2\eta_2\cos\theta_i}{\ldots}$ (Fresnel's Transmission Coefficients)

$$\eta_2 \cos\theta_i + \eta_1 \cos\theta_t$$

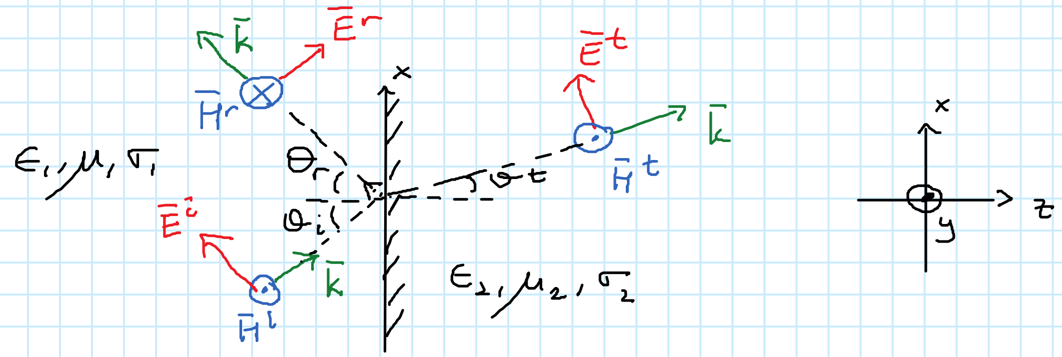
$$\boxed{T_{TE} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}}$$

(Fresnel's Transmission Coefficients)

P24

Monday, April 26, 2021 2:51 PM

b-) Parallel (TM) Polarization



where

$$\bar{E}^i = (\hat{a}_x \cos\theta_i - \hat{a}_z \sin\theta_i) E_o e^{-j k_1 (x \sin\theta_i + z \cos\theta_i)}$$

$$\bar{H}^i = \hat{a}_y \frac{E_o}{\eta} e^{-j k_1 (x \sin\theta_i + z \cos\theta_i)}$$

and

$$\bar{E}^r = (\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r) \Gamma_{TM} E_o e^{-j k_1 (x \sin\theta_r - z \cos\theta_r)}$$

$$\bar{H}^r = -\hat{a}_y \Gamma_{TM} \frac{E_o}{\eta_1} e^{-j k_1 (x \sin\theta_r - z \cos\theta_r)}$$

and

$$\bar{E}^t = (\hat{a}_x \cos\theta_t - \hat{a}_z \sin\theta_t) T_{TM} E_o e^{-j k_2 (x \sin\theta_t + z \cos\theta_t)}$$

$$\bar{H}^t = \hat{a}_y T_{TM} \frac{E_o}{\eta_2} e^{-j k_2 (x \sin\theta_t + z \cos\theta_t)}$$

Also, after imposing the B.C.'s:

1-) $\boxed{\theta_r = \theta_i}$ (Snell's law of reflection)

2-) $\boxed{k_1 \sin\theta_i = k_2 \sin\theta_t}$ (Snell's law of refraction)

and

$$\boxed{\Gamma_{TM} = \frac{-\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

$$\boxed{T_{TM} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}}$$

Ex:

A TE polarized plane wave is incident from air to glass interface ($\epsilon_r = 9$) with an angle of incidence 60°

Find the reflection and transmission coefficients

P25

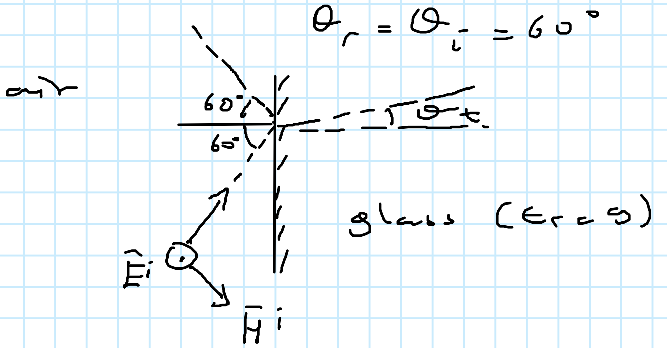
Monday, April 26, 2021 3:00 PM

Find the amount of power transfered from medium 1 to medium 2. Given $E_o = 1 \frac{mV}{m}$.

Ans:

From the Snell's law of reflection:

$$\theta_r = \theta_i = 60°$$



From the Snell's law of refraction.

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

where $k_1 = \omega\sqrt{\mu_1\epsilon_1} = \omega\sqrt{\mu_o\epsilon_o}$

and $k_2 = \omega\sqrt{\mu_2\epsilon_2} = \omega\sqrt{\mu_o\underbrace{\epsilon_{r_2}}_{9}\cdot\epsilon_o} = 3\omega\sqrt{\mu_o\epsilon_o}$

Substituting k_1 and k_2 into the Snell's law of refraction,

$$\omega\cancel{\sqrt{\mu_o\epsilon_o}}\underbrace{\sin\theta_i}_{\frac{\sqrt{3}}{2}} = 3\omega\cancel{\sqrt{\mu_o\epsilon_o}}\sin\theta_t$$

$\Rightarrow \sin\theta_t = \frac{\sqrt{3}}{6} \quad \Rightarrow \theta_t = \sin^{-1}\left(\frac{\sqrt{3}}{6}\right) = 16.78° \approx 17°$

Also,

$$\Gamma_{TE} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

where

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_o}{\epsilon_o}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_o}{\epsilon_{r_2}\epsilon_o}} = \frac{1}{3}\eta_1$$

$$\Gamma_{TE} = \frac{\frac{1}{3}\cancel{\eta_1}\cos 60° - \cancel{\eta_1}\cos 17°}{\frac{1}{3}\cancel{\eta_1}\cos 60° + \cancel{\eta_1}\cos 17°} = \frac{\frac{1}{3}\cdot(0.5) - 0.9563}{\frac{1}{3}(0.5) + 0.9563}$$

$$\Gamma_{TE} = -0.7$$

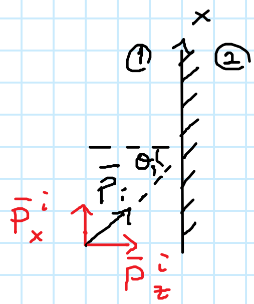
$\Rightarrow 1 + \Gamma = T \quad \Rightarrow 1 - 0.7 = T$

$$\Rightarrow T_{TE} = 0.3$$

P26

Monday, April 26, 2021 3:39 PM

Power Analysis:



$\Gamma_{TE} = -0.7$

$T_{TE} = 0.3$

$$|\bar{P}^i_{av}| = \frac{1}{2} \mathrm{Re}[\bar{E}^i \times \bar{H}^{i*}] = \frac{1}{2} \frac{E_0^2}{\eta_1} \left(\frac{W}{m^2}\right)$$

$$= \frac{1}{2} \frac{(10^{-3})^2}{377} = 1.32 \frac{nW}{m^2}$$

$$|\bar{P}^i_{z_{av}}| = |\bar{P}^i_{av}| \cdot \cos \underset{\substack{\uparrow \\ 60^\circ}}{\theta_i} = \left(1.32 \frac{nW}{m^2}\right) \cdot \left(\frac{1}{2}\right) = \boxed{0.66 \frac{nW}{m^2}}$$

$$|\bar{P}^r_{av}| = \frac{1}{2} \mathrm{Re}[\bar{E}_r \times \bar{H}^{r*}] = \frac{1}{2} \frac{E_0^2}{\eta_1} \cdot |\Gamma|^2 = \frac{1}{2} \frac{(10^{-3})^2}{377} (0.7)^2$$

$$= 0.65\, nW.$$

$$\Rightarrow |\bar{P}^r_{z_{av}}| = |\bar{P}^r_{av}| \cos 60^\circ = (0.65\, nW)(0.5) = \boxed{0.325\, nW}$$

$$|\bar{P}^t_{av}| = \frac{1}{2} \frac{E_0^2}{\eta_1} \cdot \frac{\eta_1}{\eta_2} \cdot |T_{TE}|^2 = (1 - |\Gamma|^2) |\bar{P}^i_{av}| = (1 - 0.49)\left(1.32 \frac{nW}{m^2}\right)$$

$$= 0.3564 \frac{nW}{m^2}$$

$$\Rightarrow |\bar{P}^t_{z_{av}}| = \left(0.3564 \frac{nW}{m^2}\right) \cdot \cos(\underset{\substack{\Downarrow \\ 17^\circ}}{\theta_t})$$

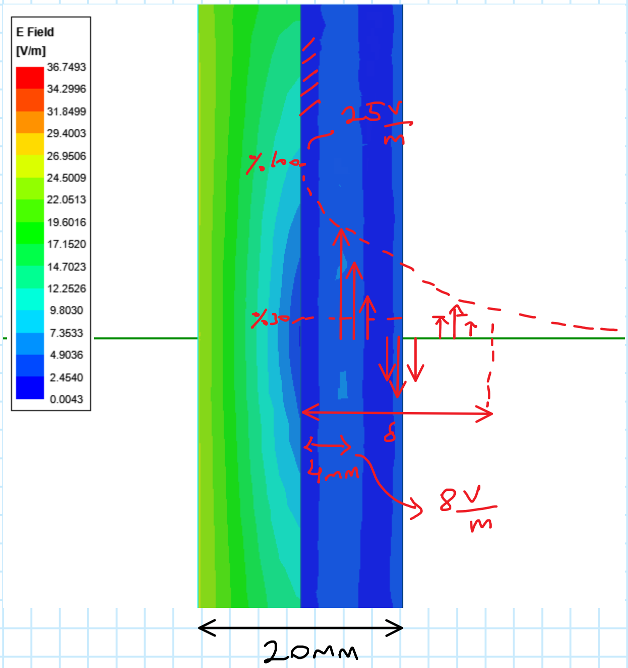
$$= \left(0.3564 \frac{nW}{m^2}\right)(0.9563) = \boxed{0.3412 \frac{nW}{m^2}}$$

$$0.325 \frac{nW}{m^2} + 0.3412 \frac{nW}{m^2} = 0.66 \frac{nW}{m^2}$$

P27

Monday, May 3, 2021 2:45 PM

Simulation of air to water interface (normal incidence at $f = 2.4\,GHz$)



Analysis:

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{(2\pi \times 2.4\times10^{9})(4\pi\times10^{-7})(5)}{2}}$$

$$= 217.65$$

$$\Rightarrow \delta = \frac{1}{\alpha} = \frac{1}{217.65} = 4.59\ mm$$

$\Rightarrow$ The simulation result is in good agreement with the calculated values.

2 Concepts:

1-) Total Transmission:

a-) TE Polarization

$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0$$

$$\eta_2 \cos\theta_i - \eta_1 \cos\theta_t = 0$$

or

$$\eta_2 \cos\theta_i = \eta_1 \cos\theta_t$$

or

$$\cos\theta_i = \frac{\eta_1}{\eta_2}\cos\theta_t \quad , \quad \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \quad \text{and} \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \ (\text{dielec.})$$

or

$$\cos\theta_i = \sqrt{\frac{\mu_1}{\mu_2}\left(\frac{\epsilon_2}{\epsilon_1}\right)} \cdot \cos\theta_t \qquad (1)$$

P28

Monday, May 3, 2021 3:07 PM

We have also, Snell's law of refraction

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

or

$$\sin\theta_i = \frac{k_2}{k_1}\sin\theta_t$$

or

$$\sin^2\theta_i = \left(\frac{k_2}{k_1}\right)^2 \sin^2\theta_t \quad , \quad k_1 = \omega\sqrt{\mu_1\epsilon_1} \quad , \quad k_2 = \omega\sqrt{\mu_2\epsilon_2}$$

or

$$\sin^2\theta_i = \left(\frac{\cancel{\omega}\sqrt{\mu_2\epsilon_2}}{\cancel{\omega}\sqrt{\mu_1\epsilon_1}}\right)^2 \sin^2\theta_t$$

or

$$\sin^2\theta_i = \frac{\mu_2}{\mu_1}\frac{\epsilon_2}{\epsilon_1}\cdot\sin^2\theta_t$$

or

$$\sin^2\theta_t = \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_i \quad \text{——(2)}$$

Then, we have

$$\cos\theta_i = \sqrt{\frac{\mu_1}{\mu_2}\left(\frac{\epsilon_2}{\epsilon_1}\right)}\cdot\cos\theta_t \quad \text{——(1)}$$

Take the square of both sides of (1).

$$\cos^2\theta_i = \frac{\mu_1}{\mu_2}\frac{\epsilon_2}{\epsilon_1}\cos^2\theta_t$$

or

$$1-\sin^2\theta_i = \frac{\mu_1}{\mu_2}\frac{\epsilon_2}{\epsilon_1}\left(1-\underline{\sin^2\theta_t}\right)$$

replace this term by eqn. (2)

Then,

$$1-\sin^2\theta_i = \frac{\mu_1}{\mu_2}\frac{\epsilon_2}{\epsilon_1}\left(1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_i\right)$$

Solving for $\sin\theta_i$ gives

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1}-\frac{\mu_2}{\mu_1}}{\frac{\mu_1}{\mu_2}-\frac{\mu_2}{\mu_1}}}$$

For θ_i to exist, the term inside the square root must be less than 1, i.e

P29

Monday, May 3, 2021 3:31 PM

$$\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1} \leq \frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1}$$

or

$$\boxed{\frac{\epsilon_2}{\epsilon_1} \leq \frac{\mu_1}{\mu_2}} \quad \text{must hold}$$

If $\mu_1 = \mu_2 = \mu_0 \Rightarrow$

$$\boxed{\epsilon_2 \leq \epsilon_1}$$

However, for $\mu_1 = \mu_2 = \mu_0$:

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\underbrace{\frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1}}_{\to 0}}} = \infty$$

$\sin\theta_i = \infty$ means that there is no solution $\Rightarrow$ No such angle exists

b-) TM Pol

$$\Gamma_{TM} = \frac{-\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_i + \sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_t}{\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_i + \sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_t} = 0$$

If we go through a similar analysis as before

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}}}$$

$$\Rightarrow \cancel{\frac{\epsilon_2}{\epsilon_1}} - \frac{\mu_2}{\mu_1} \leq \cancel{\frac{\epsilon_2}{\epsilon_1}} - \frac{\epsilon_1}{\epsilon_2} \Rightarrow \boxed{\frac{\mu_2}{\mu_1} \geq \frac{\epsilon_1}{\epsilon_2}} \quad \text{must hold.}$$

If $\mu_1 = \mu_2 = \mu_0 \Rightarrow$

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - 1}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}}} = \sqrt{\frac{\frac{\epsilon_2 - \epsilon_1}{\epsilon_1}}{\frac{\epsilon_2^2 - \epsilon_1^2}{\epsilon_1\epsilon_2}}}$$

or

$$\sin\theta_i = \left[\frac{\cancel{\epsilon_2 - \epsilon_1}}{\cancel{\epsilon_1}} \cdot \frac{\cancel{\epsilon_1}\epsilon_2}{(\cancel{\epsilon_2 - \epsilon_1})(\epsilon_2 + \epsilon_1)}\right]^{\frac{1}{2}} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\Rightarrow \boxed{\theta_i = \sin^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}\right)}$$

(Angle of total transmission or Brewster angle)

P30

Monday, May 3, 2021 3:42 PM

Also it can be written as

$$\boxed{\theta_i = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)}$$

- Thus, the 2^{nd} medium must be denser.

$$1 = \frac{\mu_0}{\mu_0} \geq \frac{\epsilon_1}{\epsilon_2}$$ ↳ in terms of polarization

$$\Rightarrow \boxed{\epsilon_2 \geq \epsilon_1}$$

Ex:

A TM pol. plane wave is incident from air to polystere ($\epsilon_r = 2.56$) interface with oblique incidence. Find the angle of incidence such that all the wave energy is transmitted.

Ans.

$\epsilon_1 = \epsilon_0$, $\epsilon_2 = 2.56\,\epsilon_0$.

$$\Rightarrow \theta_i = \sin^{-1}\left(\sqrt{\frac{2.56\,\cancel{\epsilon_0}}{\cancel{\epsilon_0}(1+2.56)}}\right) = \sin^{-1}\left(\sqrt{\frac{2.56}{3.56}}\right) = 58^\circ //.$$

Alternatively, $\theta_i = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \tan^{-1}\left(\sqrt{\frac{2.56\,\cancel{\epsilon_0}}{\cancel{\epsilon_0}}}\right) = 58^\circ //.$

2-) Total Reflection

a-) TE Pol.

$$|\Gamma_{TE}| = \left|\frac{\sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t}\right| = 1 \qquad (1)$$

Also,

$$\cos\theta_t = \sqrt{1-\sin^2\theta_t} = \sqrt{1 - \frac{\mu_1}{\mu_2}\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}$$

Snell's law of refraction

For θ_t to not exist

$$\frac{\mu_1}{\mu_2}\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i \geq 1$$

This condition makes $\cos\theta_t$ complex and no solution exists for θ_t

P31

Monday, May 3, 2021 4:02 PM

$$\frac{\mu_1}{\mu_2}\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i \geq 1$$

Solving for θ_i gives

$$\boxed{\theta_i \geq \sin^{-1}\left(\sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}\right)}$$ (condition for all reflection.)

for when $\theta_i = \sin^{-1}\left(\sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}\right)$ is called "critical angle".

The condition

$$\boxed{\mu_2\epsilon_2 \leq \mu_1\epsilon_1}$$ to θ_i to exist

For $\mu_1 = \mu_2 = \mu_0$

$$\boxed{\theta_i = \sin^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)}$$ critical angle

and the condition becomes

$$\boxed{\epsilon_2 \leq \epsilon_1}$$

– The 1^{st} medium must be denser

b-) TM pol.

The critical angle formulations and equations and the conditions are the same as in TE pol.

– Thus, critical angle is polarization independent

Ex:

A TE pol. wave is incident at glass to air interface ($\epsilon_r = 9$ for glass). Find the angle for all waves to be reflected.

Ans.

$\epsilon_1 = 9\epsilon_2$, $\epsilon_2 = \epsilon_0$

$$\Rightarrow \theta_i = \theta_c = \sin^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \sin^{-1}\left(\sqrt{\frac{1}{9}}\right) = \sin^{-1}\left(\frac{1}{3}\right) \approx 20^\circ.$$

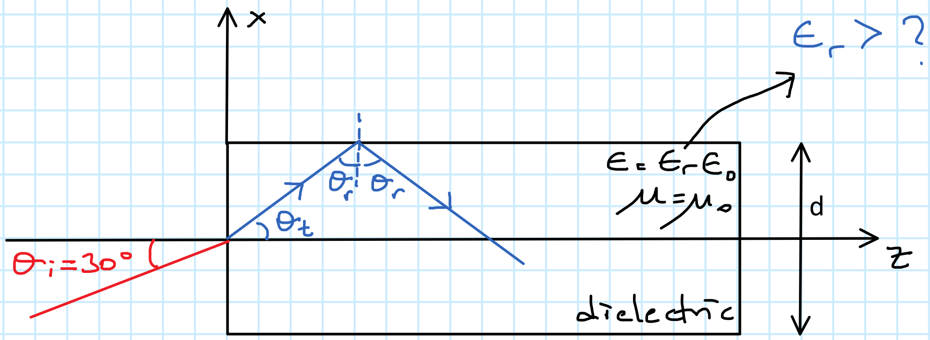
If $\theta_i \geq \theta_c \Rightarrow$ all energy reflects.

P32

Monday, May 3, 2021 4:42 PM

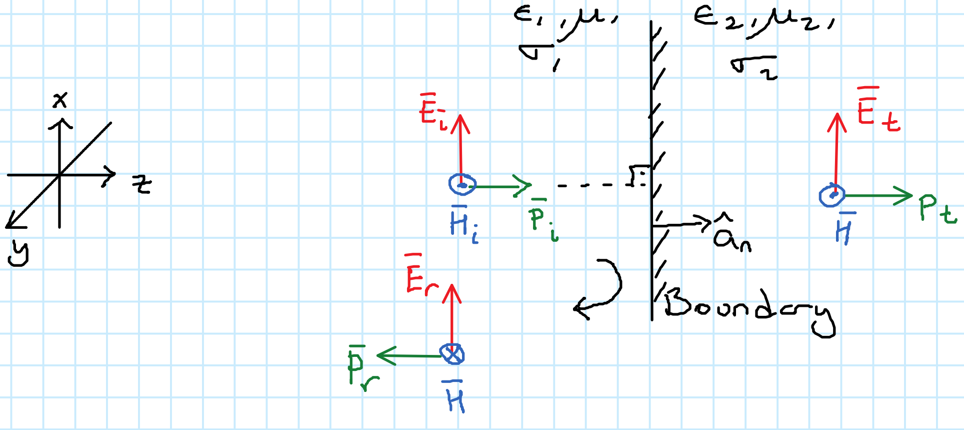
HW #7:

For a given dielectric cylindrical structure with diameter d, the plane wave enters the structure from air from its one end. For $\theta_i = 30°$, find the required range of dielectr. constant ϵ_r for the dielectric material so that the energy is retained inside the structure.



– Normal and Oblique Incidences in Lossy Media –

– Normal Incidence:



Writing the expressions for the fields

$$\bar{E}_i = \hat{a}_x E_0 e^{-\alpha_1 z} e^{-j\beta_1 z} \quad \text{(Phasor)}$$

$$\bar{H}_i = \hat{a}_y \frac{E_0}{\eta_1} e^{-\alpha_1 z} e^{-j\beta_1 z}$$

and

$$\bar{E}_r = \hat{a}_x \Gamma . E_0 \, e^{+\alpha_1 z} e^{j\beta_1 z}$$

$$\bar{H}_r = -\hat{a}_y \Gamma . \frac{E_0}{\eta} e^{\alpha_1 z} e^{j\beta_1 z}$$

Note. For lossy media,

$\gamma = \alpha + j\beta$ ← Phase constant

For lossless media,

$\alpha = 0$, k = wavenumber = β

P33

Monday, May 10, 2021 2:31 PM

and

$$\bar{E}_t = \hat{a}_x T \cdot E_o e^{-\alpha_2 z} \cdot e^{-j\beta_2 z} \Rightarrow \bar{E}_t(z,t) = \hat{a}_x T E_o e^{-\alpha_2 z} \cos(\omega t - \beta_2 z)$$

$$\bar{H}_t = \hat{a}_y T \frac{E_o}{\eta_2} e^{-\alpha_2 z} \cdot e^{-j\beta_2 z}$$

and

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$, where η_1, η_2 are complex from the table in pg. 17.

$$T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Power Analysis

$$S^i_{avg} = \hat{a}_z \frac{|E_o|^2}{2} e^{-2\alpha_1 z} \, \mathrm{Re}\left(\frac{1}{\eta_1^*}\right)$$

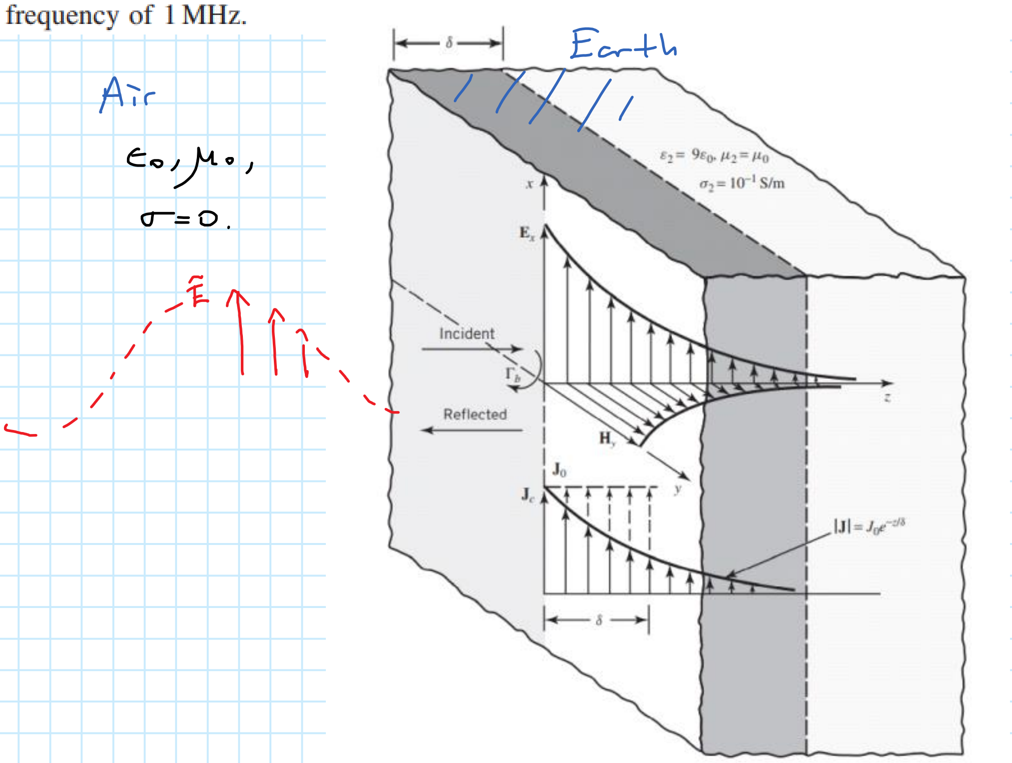
$$S^r_{avg} = -\hat{a}_z |\Gamma|^2 \frac{|E_o|^2}{2} e^{+2\alpha_1 z} \, \mathrm{Re}\left(\frac{1}{\eta_1^*}\right)$$

$$S^t_{avg} = \hat{a}_z |T|^2 \frac{|E_o|^2}{2} e^{-2\alpha_2 z} \, \mathrm{Re}\left(\frac{1}{\eta_2^*}\right)$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta_1$ and η_2 can be computed from the table in pg 17 with constants $\epsilon_1, \epsilon_2, \mu_1, \mu_2$ and σ_1 and σ_2 are given.

Ex

A uniform plane wave, whose incident electric field has an x component with an amplitude at the interface of 10^{-3} V/m, is traveling in a free-space medium and is normally incident upon a lossy flat earth as shown in Figure ~~5.4~~. Assuming that the constitutive parameters of the earth are $\varepsilon_2 = 9\varepsilon_0$, $\mu_2 = \mu_0$ and $\sigma_2 = 10^{-1}$ S/m, determine the variation of the conduction current density in the earth at a frequency of 1 MHz.



P34

Monday, May 10, 2021 2:44 PM

Ans

- Find $\bar{E}_t$
- $\bar{J}_c = \sigma_2 \cdot \bar{E}_t$ (Ohm's law)

$\Rightarrow$ We need to evaluate $T = \dfrac{2\eta_2}{\eta_2 + \eta_1}$ where $\eta_1 = 377\,\Omega$ and

to find η_2:

We check: $\left(\dfrac{\sigma_2}{\omega \epsilon_2}\right)^2 = \left(\dfrac{10^{-1}}{2\pi(10^6)9.(8.854\times10^{-12})}\right)^2 = 40000 \gg 1. \Rightarrow$ Good conductor

$$\Rightarrow \eta_2 = \sqrt{\frac{\omega\mu}{2\sigma}}\,(1+j) = \sqrt{\frac{(2\pi\times10^6)(4\pi\times10^{-7})}{2\times10^{-1}}}\,(1+j) = 2\pi(1+j)$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\pi(1+j) - 377}{2\pi(1+j) + 377} = 0.967\angle 178.1^\circ$$

and

$$T = 1 + \Gamma = 1 + 0.967\angle 178.1^\circ$$

Thus,

$$\bar{E}_t = \hat{a}_x T \cdot E_o e^{-\alpha_2 z} \cdot e^{-j\beta_2 z}$$
$$\bar{H}_t = \hat{a}_y T \frac{E_o}{\eta_2} e^{-\alpha_2 z} \cdot e^{-j\beta_2 z}$$

where $E_o = 1 \frac{mV}{m}$.

$$\Rightarrow |\bar{E}_t| = |T| \cdot |E_o| = 10^{-3} |0.0335 + j0.0321| = 4.64\times10^{-5} \frac{V}{m}$$

$$\bar{J}_c \Big|_{z=0} = \sigma |\bar{E}_t| = 10^{-1} \frac{s}{m}\left(4.64\times10^{-5} \frac{V}{m}\right) = 4.64\times10^{-6} \frac{V.s}{m^2}$$

$$\Rightarrow J_o = 4.64 \,\mu \frac{A}{m^2}$$

$$|\bar{J}_c(z)| = J_o \cdot e^{-\alpha_2 z} = J_o\, e^{-\frac{z}{\delta}}$$

where

$$\delta = \sqrt{\frac{2}{\omega\mu_2\sigma_2}} = \sqrt{\frac{2}{2\pi\times10^6(4\pi\times10^{-7})10^{-1}}} = 1.59\,m.$$

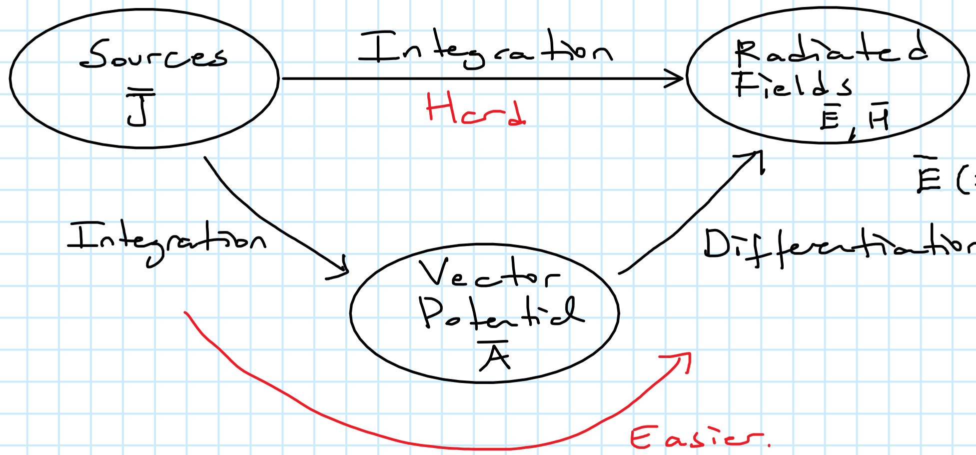
$$\Rightarrow |\bar{J}_c(z)| \Big|_{z=1.59\,m} = J_o \cdot e^{-1} = 0.3679 \cdot J_o = 0.3679\left(4.64\,\mu\frac{A}{m^2}\right) = 1.707\,\frac{\mu A}{m^2}$$

- Oblique Incidence:

The formulations of the oblique incidence in lossy media are the same as that of the lossless medium with α, β, η and other parameters are functions of σ as in table in p5-9.

- Radiation -

The problem of radiation can be depicted as in the figure below



$$\bar{E}(z) = \hat{a}_x E_o e^{-jkz}$$

Differentiation Path:

$$\boxed{\bar{E} = -j\omega\bar{A} - j\frac{1}{\omega\mu\epsilon}\bar{\nabla}(\bar{\nabla}\cdot\bar{A})}$$

Integration Path:

$$\boxed{\bar{A} = \frac{\mu}{4\pi}\int_V \bar{J}\cdot\frac{e^{-jkR}}{R}\,dv'} \quad , \quad \bar{J} = \frac{A}{m^2} \quad \text{(volumetric source)}$$

or for surface currents (surface source)

$$\boxed{\bar{A} = \frac{\mu}{4\pi}\int_S \bar{J}_s\cdot\frac{e^{-jkR}}{R}\,ds'} \quad , \quad \bar{J}_s = \frac{A}{m} \quad \text{(surface source)}$$

or for linear current sources:

$$\boxed{\bar{A} = \frac{\mu}{4\pi}\int_C \bar{I}\cdot\frac{e^{-jkR}}{R}\,dl'} \quad , \quad I = A \quad \text{(current)}$$

$$\boxed{\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} \, dl'}$$, $I = A$ (current)

P36

Monday, May 10, 2021 3:46 PM

Far field approximations $(R \gg L)$

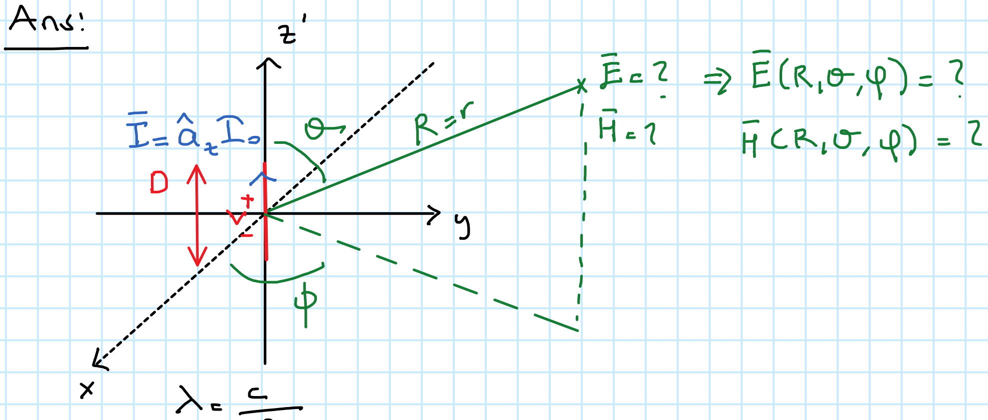
$\boxed{R = r}$ (radial distance from the origin)

$\boxed{\bar{E} = -j\omega \bar{A}}$ for E_θ component only $E_R = 0$

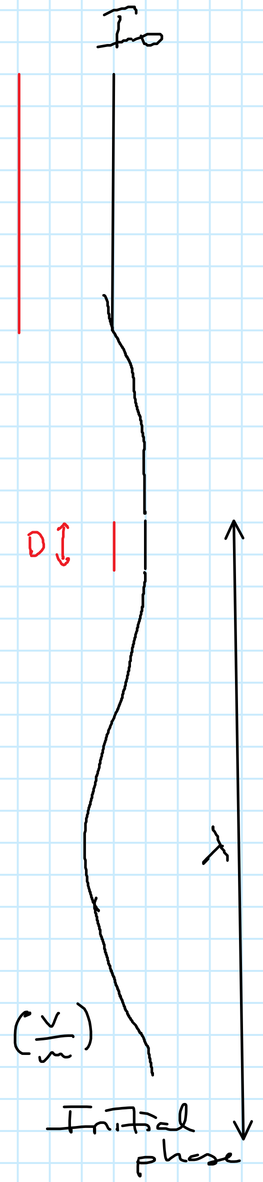
Ex:

Given a very small conductor $(D \ll \lambda)$ with a current $\bar{I} = \hat{a}_z I_o$ (phasor), find the radiated fields in air?

Ans:



$\lambda = \frac{c}{f}$



To find A:

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} \, dl'$$

or

$$\bar{A} = \frac{\mu_o}{4\pi} \int_C (\hat{a}_z I_o) \frac{e^{-jkr}}{r} \, dz'$$

$$\Rightarrow \bar{A} = \frac{\cancel{4\pi} \times 10^{-7}}{\cancel{4\pi}} \int_{-D/2}^{D/2} \hat{a}_z I_o \frac{e^{-jkr}}{r} \, dz'$$

$A_\theta = A_z \sin\theta$

$$\Rightarrow \bar{A} = \hat{a}_z 10^{-7} . I_o . \frac{e^{-jkr}}{r} D .$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}_\theta \Rightarrow \bar{E}_\theta = -j\omega 10^{-7} . I_o \frac{e^{-jkr}}{r} . D . \sin\theta \quad \left(\frac{V}{m}\right)$$

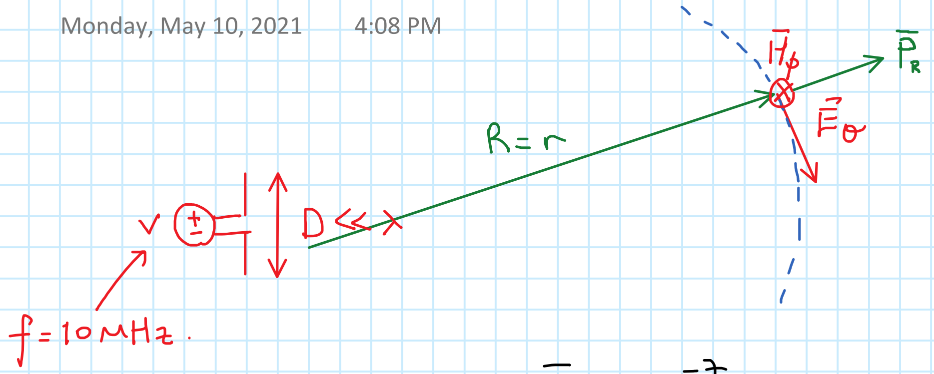
or

$$E_\theta = 10^{-7} \omega I_o . D . \sin\theta \frac{e^{-jkr}}{r} e^{-j\frac{\pi}{2}}$$

$$E_\theta(r, t) = Re\left[E_\theta . e^{j\omega t}\right] = \underbrace{10^{-7} \omega I_o . D . \frac{1}{r}}_{E_o} \sin\theta \cos\left(\omega t - \underbrace{kr}_{\text{Phase factor}} - \frac{\pi}{2}\right)$$

P37

Monday, May 10, 2021 4:08 PM



$$\lambda\Big|_{f=10\,MHz} = \frac{c}{f} = \frac{3\times 10^{8}}{10\times 10^{6}} = 30$$

$D \ll 30\,m$. $D = 1\,m$ ✓

$$\bar{E}_\theta = 10^{-7}\,\omega\, I_o\,.D.\,\frac{e^{-jkr}}{r}\; e^{-j\frac{\pi}{2}} \quad \text{(Phasor)}$$

$$E_\theta = \underbrace{10^{-7}\,\omega\, I_o\,.D.\,\frac{1}{r}}_{E_o}\;\cos\big(\omega t \underbrace{- k r - \tfrac{\pi}{2}}_{\text{phase factor}}\big)$$

$$\Rightarrow \bar{H}(r) = \frac{1}{\eta}\,\hat{a}_n \times \bar{E}(r)$$

$$= \frac{1}{377}\,.\,\hat{a}_R \times \bar{E}_\theta$$

$$= \hat{a}_\phi\,\frac{10^{-7}\,\omega\, I_o\, D}{377}\,\frac{1}{r}\,\cos\!\left(\omega t - kr - \frac{\pi}{2}\right)$$

$\bar{P} = \bar{E} \times \bar{H}$ (W/m^2) $\Rightarrow$ in $\hat{a}_R$ direction

Ex:

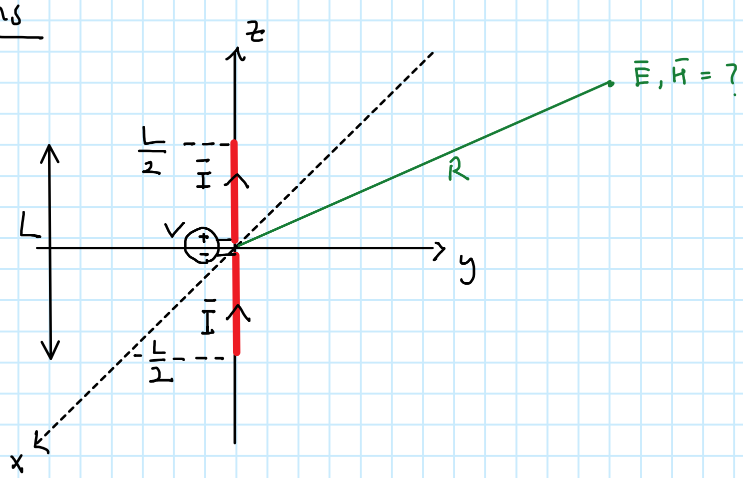
Finite length conductor is used

The current density is given as

$$\bar{I}(z') = \begin{cases} \hat{a}_z\, I_o \sin\left[k\left(\frac{L}{2} - z'\right)\right], & 0 \le z' \le L/2 \\ \hat{a}_z\, I_o \sin\left[k\left(\frac{L}{2} + z'\right)\right], & -\frac{L}{2} \le z' \le 0 \end{cases}$$

Find the radiated fields at far away from the conductor. frequency = 1 GHz.

Ans



P38

Monday, May 17, 2021 2:46 PM

Analysis of $I(z')$

$$\bar{I}(z') = \begin{cases} \hat{a}_z I_o \sin\left[k\left(\frac{L}{2} - z'\right)\right], & 0 \leq z' \leq L/2 \\ \hat{a}_z I_o \sin\left[k\left(\frac{L}{2} + z'\right)\right], & -\frac{L}{2} \leq z' \leq 0 \end{cases}$$

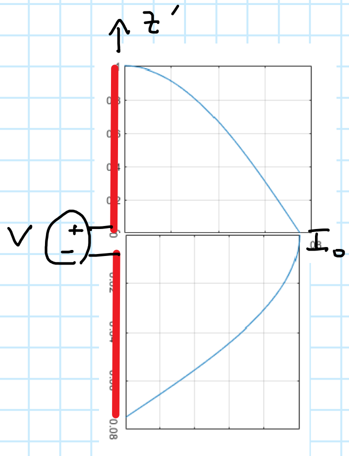
$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^9} = 0.3 = 30\,\text{cm}$$

$$k = \text{wave number} = \frac{2\pi}{\lambda} = \frac{2\pi}{0.3} = \frac{2\pi}{\frac{3}{10}} = \frac{20\pi}{3}$$

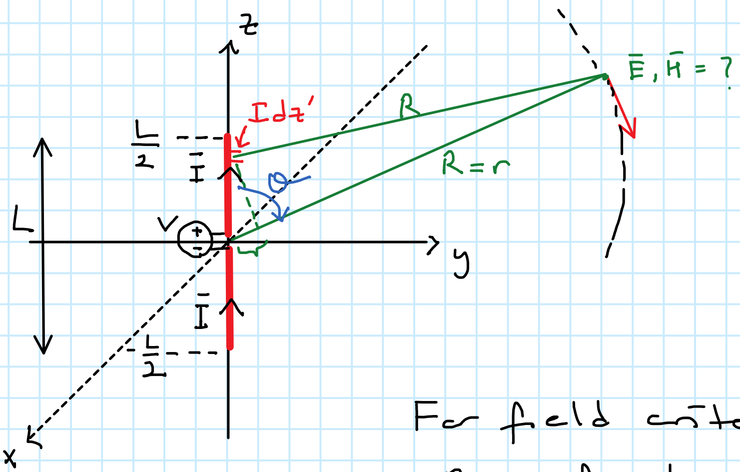
$$L = \text{Conductor length} = \frac{\lambda}{2} = 15\,\text{cm} \quad (\text{Half-wave dipole})$$

$$\Rightarrow \bar{I}(z') = \begin{cases} \hat{a}_z I_o \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{2} - z'\right)\right], & 0 \leq z' \leq L/2 \\ \hat{a}_z I_o \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{2} + z'\right)\right], & -\frac{L}{2} \leq z' \leq 0 \end{cases}$$

(Phasor)



Finding $\bar{A}$ from $\bar{I}(z')$



$$\bar{A} = \frac{\mu}{4\pi}\int_C \bar{I}\cdot\frac{e^{-jkR}}{R}\,dl'$$

$$\Rightarrow \bar{A} = \frac{\mu}{4\pi}\int_0^{L/2} \bar{I}(z')\frac{e^{-jkR}}{R}\,dz' + \frac{\mu}{4\pi}\int_{-\frac{L}{2}}^{0} \bar{I}(z')\frac{e^{-jkR}}{R}\,dz'$$

For field criteria: $R \gg L$.

$R = r$ for denominator term

$R = r - z'\cos\theta$ for the phase term

The integral gives $\bar{A} = \hat{a}_z \eta \frac{I_o e^{-jkr}}{\omega 2\pi r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right]$

P39

Monday, May 17, 2021 3:23 PM

and $\bar{E} = -j\omega A$

$$\Rightarrow \boxed{E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] \left(\frac{V}{m}\right)}$$

$$H_\phi = j \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] \left(\frac{A}{m}\right)$$

$$\bar{P}_{avg} = \hat{a}_R \left(\eta \frac{I_o}{2\pi r}\right)^2 / 2\eta \left(\frac{W}{m^2}\right) \qquad \bar{P}_{avg} = \frac{|E_o|^2}{2\eta}$$

↳ This is the average power

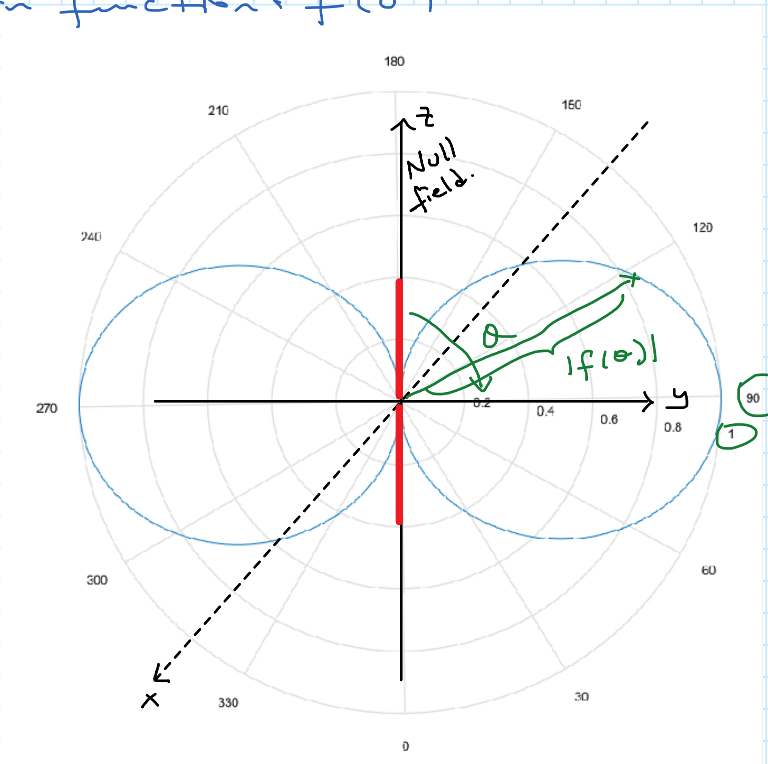
assuming that

$$f(\theta) = \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] = 1 \text{ (maximum)}$$

where $f(\theta)$ is the part of the field containing angular dependence.

$\Rightarrow f(\theta)$ = Pattern function (Radiation pattern function)

Pattern function: $f(\theta)$



$f(\theta)$ for $L = \frac{\lambda}{2}$ (Half-wave dipole)

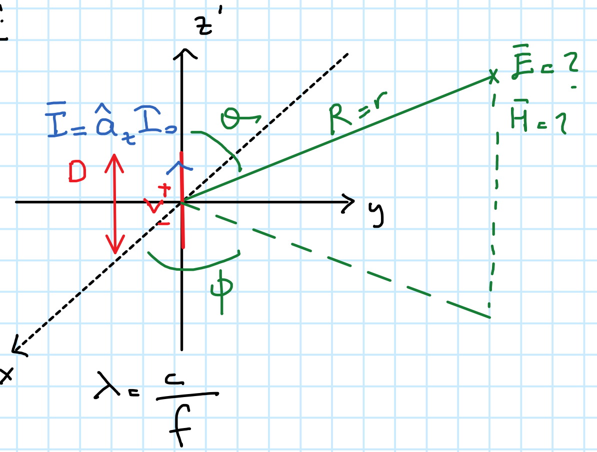
P40

Wednesday, July 28, 2021 11:47 AM

Ex:

Given a very small conductor ($D \ll \lambda$) with a current $\bar{I} = \hat{a}_z I_o z^2$ (phasor), find the radiated fields in air ?

Ans:



$\Rightarrow \bar{E}(R, \theta, \varphi) = ?$

$\bar{H}(R, \theta, \varphi) = ?$

$\lambda = \dfrac{c}{f}$

To find A :

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} \, dl'$$

or

$$\bar{A} = \frac{\mu_o}{4\pi} \int_C (\hat{a}_z I_o) \frac{e^{-jkr}}{r} \, dz'$$

$$\int_{-\frac{D}{2}}^{\frac{D}{2}} z^2 \, dz = \frac{1}{3} z^3 \Big|_{-\frac{D}{2}}^{D/2} = \frac{1}{12} D^3$$

$$\Rightarrow \bar{A} = \frac{\cancel{4\pi} \times 10^{-7}}{\cancel{4\pi}} \int_{-D/2}^{D/2} \hat{a}_z I_o z^2 \frac{e^{-jkr}}{r} \, dz'$$

$$\Rightarrow \bar{A} = \hat{a}_z 10^{-7} \cdot I_o \cdot \frac{e^{-jkr}}{r} \frac{D^3}{12}$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}_\theta \Rightarrow \bar{E}_\theta = -j\omega 10^{-7} \cdot I_o \frac{e^{-jkr}}{r} \cdot \frac{D^3}{12} \sin\theta \left(\frac{V}{m}\right)$$

or

$$E_\theta = 10^{-7} \omega I_o \cdot D \cdot \sin\theta \frac{e^{-jkr}}{r} e^{-j\frac{\pi}{2}}$$

$$\bar{E}_\theta(r,t) = Re\left[E_\theta \cdot e^{j\omega t}\right] = \underbrace{10^{-7} \omega I_o \frac{D^3}{12} \cdot \frac{1}{r} \cdot \overbrace{\sin\theta}^{f(\theta) = \text{Pattern function}}}_{E_o} \cdot \cos(\omega t \underbrace{- kr - \tfrac{\pi}{2}}_{\text{phase factor}})$$

P41

Wednesday, July 28, 2021 12:01 PM

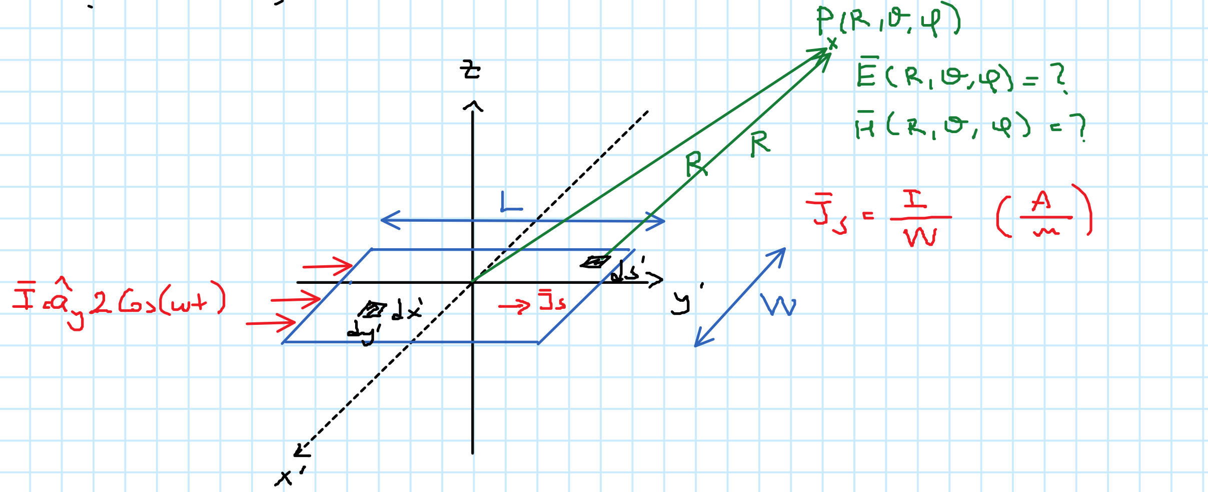
Midterm Exam Information:

- Midterm exam will be held on 06/08/2021 Friday at 13:30 usual lecture time.
- The exam will be open notes.
- Mobile phones and computers are not allowed to be used during the exam. However, you may use a standart calculator.
- The exam will be online with open-camera.
- During the exam, your hands and paper must be visible on the screen.

Ex:

Given a rectangular small ($D \ll \lambda$) sheet with dimensions W and L, a current of 2A at f=1GHz is fed from its one end. Find the radiated $\bar{E}$ and $\bar{H}$ fields? (W=2, L=2)

Ans:



$$\Rightarrow \bar{J}_s = \hat{a}_y \frac{I}{W} = \hat{a}_y\, 1 \left(\frac{A}{m}\right)$$

First, find $\bar{A}$ from $\bar{J}_s$:

$$\bar{A} = \frac{\mu}{4\pi} \int_S \bar{J}_s \cdot \frac{e^{-jkR}}{R}\, ds' \quad \hookrightarrow dx'dy'$$

If $\boxed{R \gg D = WL}$

$\Rightarrow R = r$ (constant)

$$\Rightarrow \bar{A} = \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{W}{2}}^{\frac{W}{2}} \hat{a}_y \cdot 1 \cdot \frac{e^{-jkr}}{r}\, dx'\, dy'$$

P42

Wednesday, July 28, 2021 12:53 PM

$$\bar{A} = \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} \hat{a}_y \cdot 1 \cdot \frac{e^{-jkr}}{r}\, dx'\, dy'$$

$$\Rightarrow \bar{A} = \hat{a}_y \frac{\mu_0}{4\pi} \frac{e^{-jkr}}{r} \underbrace{\int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} dx'\, dy'}_{w \cdot L = \cancel{4}\, m^2}$$

$$\bar{A} = \hat{a}_y \underbrace{\frac{\mu_0}{\pi} \frac{e^{-jkr}}{r}}_{A_y}$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}$$

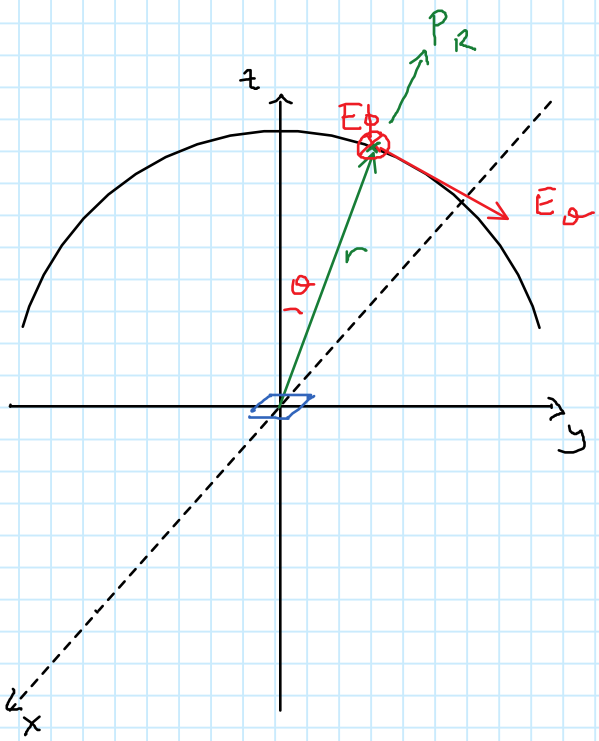
↑ θ or ϕ coord.

Thus, $\boxed{A_\theta = A_y \cos\theta \sin\phi} = \frac{\mu_0}{\pi} \frac{e^{-jkr}}{r} \cos\theta \sin\phi$, $\boxed{A_\phi = A_y \cos\phi}$ $= \frac{\mu_0}{\pi} \frac{e^{-jkr}}{r} \cos\phi$.

$$\Rightarrow E_\theta = -j\omega A_\theta \quad , \quad E_\phi = -j\omega A_\phi$$

$$E_\theta = \frac{\mu_0}{\pi} \underbrace{\cos\theta \sin\phi}_{f(\theta,\varphi)} \frac{e^{-jkr}}{r} \left(\frac{V}{m}\right) \quad \text{(Phasor)}$$

$$E_\phi = \underbrace{\frac{\mu_0}{\pi} \underbrace{\cos\phi}_{f(\varphi)}}_{E_0} \frac{e^{-jkr}}{r} \left(\frac{V}{m}\right) \quad \text{(Phasor)}$$

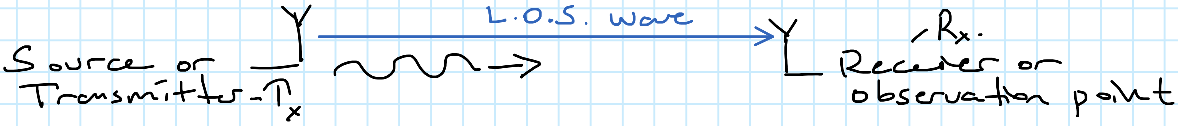


- Electromagnetic Wave Propagation -

In general, there are 3 types of wave propagation.

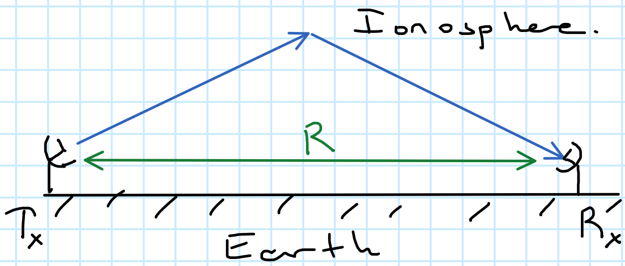
1-) Direct Wave: (Line of Sight Propagation)

- In this propagation, there are no obstacles in between the transmitter and the receiver antennas



2-) Sky Wave (Ionospheric Propagation)

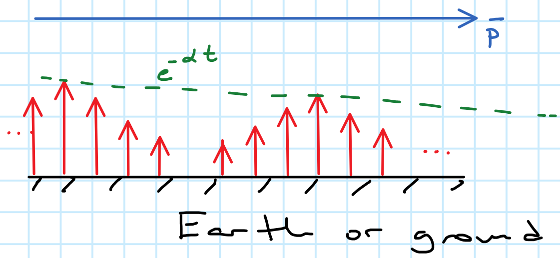
- The source is directed towards the sky, and the waves are reflected from the ionosphere



- Long range communication is possible (R is very large)
- Waves are reflected from the ionosphere

3-) Ground Wave (Surface Waves) Propagation.

- These waves can be parallel or perpendicular to ground
- Ground waves move parallel to earth surface with attenuation



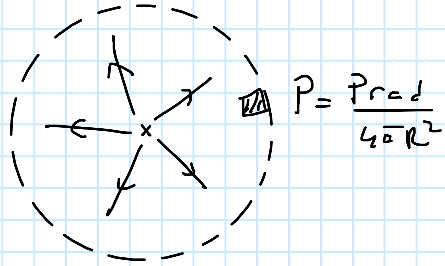
P44

Monday, May 24, 2021 3:53 PM

1-) Direct Wave Propagation:

The power density of an isotropic (equally in all directions) source is:

$$\boxed{P = \underbrace{\frac{P_{rad}}{4\pi R^2}}_{\text{Isotropic radiated power density } (W/m^2)} \left(\frac{W}{m^2}\right)}$$



P_{rad} = Total radiated power (W)

In reality the sources (antennas) are not isotropic. They radiate more power in certain directions than the others.

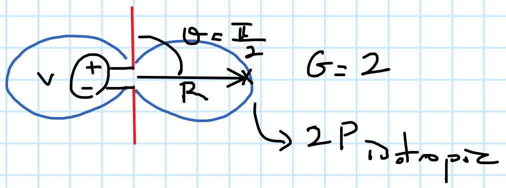
Power density of such a source is:

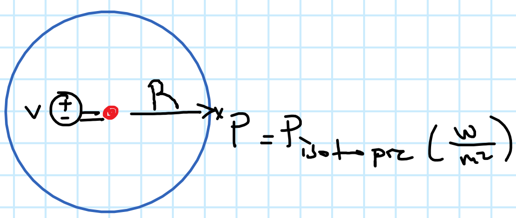
$$\boxed{P(\theta, \varphi) = \underbrace{\frac{P_{rad}}{4\pi R^2}}_{P_{iso} = \text{Isotropic rad power density}} \cdot \underbrace{G(\theta, \varphi)}_{\text{Gain of the source in } \theta, \varphi \text{ direction}} \left(\frac{W}{m^2}\right)}$$

- Unless stated otherwise, θ and φ are the angles which make $G(\theta, \varphi)$ maximum

$$\boxed{\text{Antenna Gain} = \frac{P(\theta, \varphi)}{P_{isotropic}}}$$

Ex. If Gain = G = 2 means that at θ and φ for which G is maximum, the power density of the source is twice that of the isotropic antenna





P45

Monday, May 24, 2021 4:15 PM

Ex:

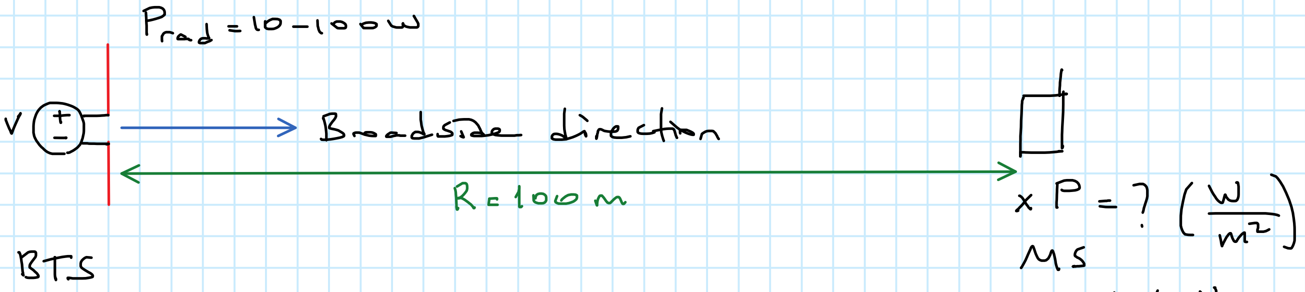
A gsm base station antenna radiates E.M. waves with 10 - 100 W of total output power. Evaluate the power density of the waves at R = 100 m away from this antenna in the broadside direction where the antenna has 12-18 dB for BTS and 2-5 dB for MS.

BTS: Base transceiver station. MS: Mobile station.

Use $G_{BTS} = 12$ dB

Note: All the given values are typical.

Ans



$$P = P_{iso} \cdot G\left(\theta = \frac{\pi}{2}\right) = \frac{P_{rad}}{4\pi R^2} \cdot G\left(\theta = \frac{\pi}{2}\right)$$

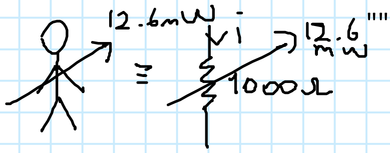
where

$$G\left(\theta = \frac{\pi}{2}\right)_{dB} = 12\ \text{dB} = 10 \log_{10} \frac{G}{G_{ref}} \quad , G_{ref} = 1$$

$$12 = 10 \log_{10} G \Rightarrow G = 10^{1.2}$$

$$\Rightarrow P = \frac{10}{4\pi(100)^2} \cdot 10^{1.2} = \boxed{1.26\ \frac{mW}{m^2}}$$

$$P = \frac{100}{4\pi(100)^2} \cdot 10^{1.2} = \boxed{12.6\ \frac{mW}{m^2}}$$

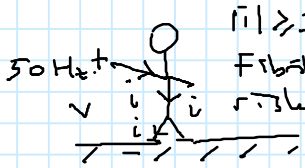


$$P_{avg} = \frac{1}{2} |i|^2 R$$

$$12.6\,mW = \frac{1}{2} \cdot |i|^2 \cdot 1000$$

$$\frac{2.6\,mW}{1000} = |i|^2$$

$$\Rightarrow |i| = 1.54\,mA.$$



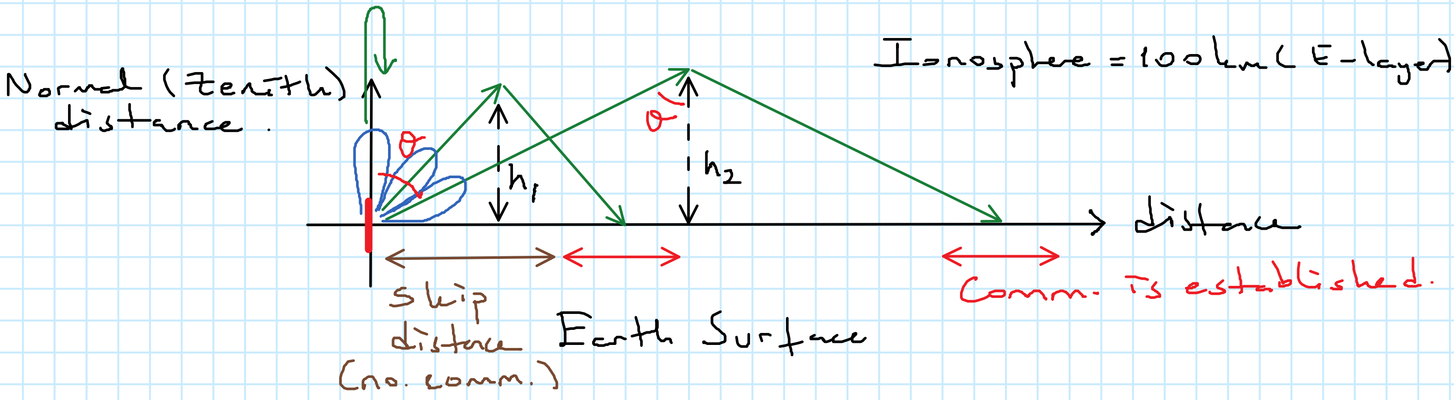
$|i| \geq 30\,mA$ Fibrilation risk!

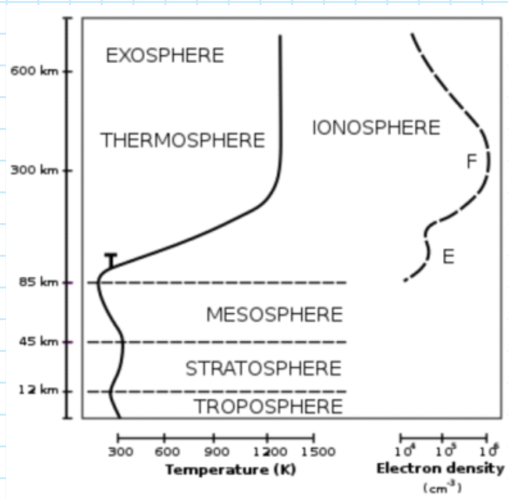
P46

Monday, May 31, 2021 2:33 PM

2-) Sky-Wave Propagation:

As waves travel through the ionosphere, they may reflect from a height h, and come back to the earth's surface at a distance d from the transmitter





Maximum Usable Frequency (MUF):

Any E.M wave whose frequency below MUF reflects from the ionosphere from a height h. Waves with frequency above MUF escape to space.

- There is also the lowest usable frequency (LUF), and optimum working frequency (OWF).

By definition:

$$\boxed{MUF = \frac{CF}{\cos\theta}}$$

where CF = Critical frequency : Max freq for reflection at zenith.

θ = Angle of incidence.

P47

Monday, May 31, 2021 2:56 PM

It is given that

$$\boxed{OWF = (0.85)\, MUF}$$ (Best freq for skywave comm.)

Also,

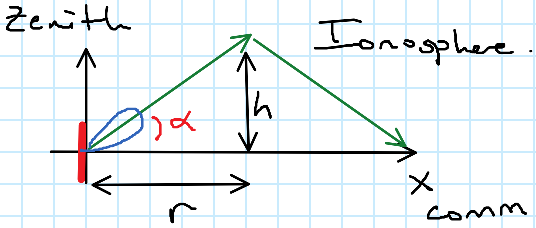
$$\boxed{CF = 9\sqrt{N_{max}}}$$

where

N_{max} = Maximum e^- density

Generally,

- Frequencies below 10MHz propagate most efficiently by skywaves at night
- Frequencies above 10MHz propagate most efficiently during the day
- At night, skip distance is increased.
- In VHF (30-300MHz), the skywave comm is not possible.
- Usually, HF (High Frequency → 3-30MHz) is used for sky wave comm.
- In attempt to calculate the radiation distance of the lowest angle beam (take-off angle), we can use the equation.



$$\tan\alpha = \frac{h}{r}$$

or

$$r = \frac{h}{\tan\alpha}$$

and the comm. distance $= \boxed{2r = \dfrac{2h}{\tan\alpha}}$, where $\alpha = 90° - \theta$

—o—

P48

Monday, May 31, 2021 3:25 PM

Ex:

Given that the maximum e^- density in the ionosphere is $N = 5\times10^4 \frac{e^-}{cm^3}$ and $\theta = 70°$. Find the MUF and OWF.

Ans.

$N_{max} = 5\times10^4 e^- . cm^{-3} = 5\times10^{10} e^- . m^{-3}$

$\Rightarrow CF = 9\sqrt{N_{max}} = 9\sqrt{5\times10^{10}} = 2\times10^6$ Hz $= 2$ MHz.

$\Rightarrow MUF = \frac{CF}{\cos\theta} = \frac{2\times10^6}{\cos 70°} = 5.884$ MHz ≈ 6 MHz.

$OWF = 0.85\ MUF = 5$ MHz.

- As a rule of thumb, MUF is about 3 times of CF.

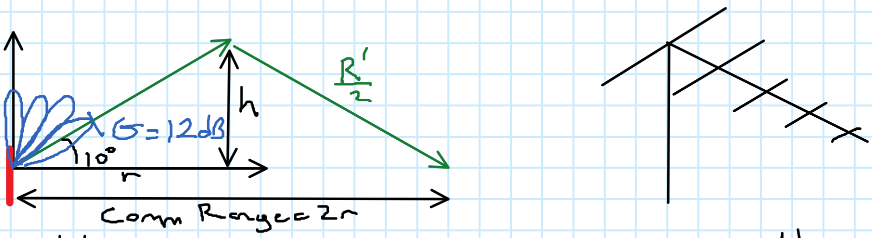
At $\theta = 30°$

$MUF = \frac{2\times10^6}{\cos 30°} = 2.3$ MHz.

- MUF $\propto$ θ (Angle of incidence)

Ex.

For a horizontal polarized log-periodic dipole antenna with 1kW of radiated power, has the following radiation pattern:



Evaluate the maximum comm range through sky waves at a single hop of the smallest angle beam, and evaluate the power density at this distance. ($h = 100$km $\to$ E-layer)

Ans.

$$Range = \frac{2h}{\tan\alpha} = \frac{2(10^5)}{\tan 10°} = \frac{200000}{0.17} = 1176.47 \text{ km.}$$

$$P = P_{iso} . Gain = P_i . G = \frac{P_{rad}}{4\pi R^2} . G = \frac{1000}{4\pi R^2}(10^{1.2}) = \frac{1000\ (10^{1.2})}{4\pi(1194000)^2}$$

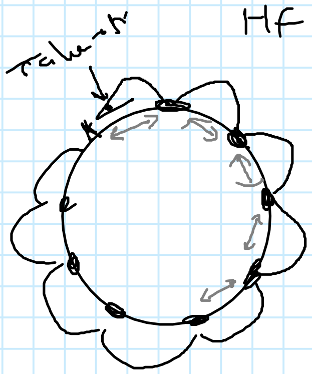
$\hookrightarrow$ range of the wave propagation $= R'$

$$\cos\alpha = \frac{r}{\frac{R'}{2}} \Rightarrow R' = \frac{1176.4 km}{\cos 10°} = \frac{1176.4 km}{\cos 10°} = 1194 \text{ km}$$

$\Rightarrow P = 8.85\times10^{-10} = 885 \frac{pW}{m^2}$

P49

Monday, May 31, 2021 3:59 PM



HF – Long wave.

– Ducting : Changes in sky-wave comm. due to seasonal and atmospheric variations

3-) Ground Wave Propagation:

The electric field expression for a ground wave:

$$E = E_0 e^{-\alpha z} e^{-j\beta z} \left(\frac{V}{m}\right)$$

$$\text{or } E(t,z) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \left(\frac{V}{m}\right) \qquad c = \frac{1}{\sqrt{\mu\varepsilon}}$$

where α = Attenuation constant

β = Phase constant. $\left(\beta = \omega\sqrt{\mu\varepsilon} = 2\pi f\sqrt{\mu\varepsilon} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}\right)$ $\left(\frac{1}{m}\right)$

Unit of α = ?

The unit of α is Nepers

Neper:

By definition, the voltage gain of the following circuit is

$\overset{+}{V_1}$ — [Circuit] — $\overset{+}{V_2}$

$$G_U = \frac{V_2}{V_1}$$

or in nepers:

$$\boxed{G_{V_{Np}} = \ln\frac{V_2}{V_1} = \ln V_2 - \ln V_1}$$

Conversion from dB to Neper:

$$G_{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 = 20\log_{10}\left(\frac{V_2}{V_1}\right)$$

Also,

$$G_{Np} = \ln\frac{V_2}{V_1} = \frac{\log_{10}\frac{V_2}{V_1}}{\log_{10} e} = \frac{\frac{G_{dB}}{20}}{\log_{10} e}$$

$$\Rightarrow \frac{G_{dB}}{G_{Np}} = 20\log_{10} e \quad \text{where } e = 2.71828\ldots$$

$$= 8.868585 \frac{dB}{Np}.$$

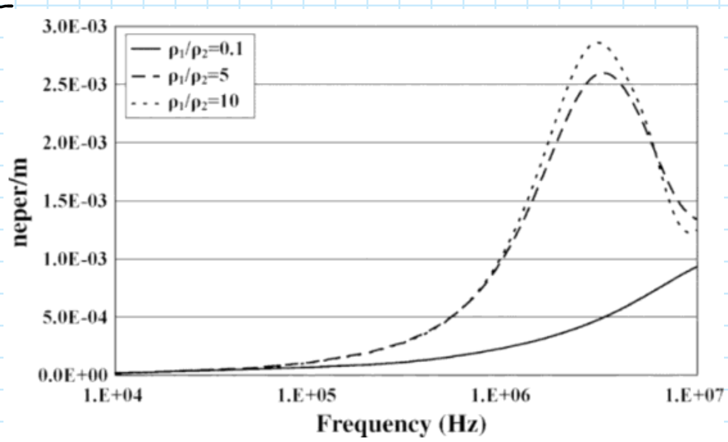
P50

Monday, May 31, 2021 4:31 PM

or

$$\frac{G_{Np}}{G_{dB}} = 0.11513 \ \frac{Np}{dB}$$

—o—



Neper Analysis

$$G_{U_{Np}} = \ln \frac{U_2}{U_1} \ (Np)$$

Let us call $\frac{U_2}{U_1} = m$.

Then,

$$G_U = \ln m$$

or

$$m = e^{G_U} \rightarrow \text{in Np.}$$

Since for a ground wave,

$$E = E_0 \underbrace{e^{-\alpha z}}_{\text{Attenuation term}} e^{-j\beta z} \left(\frac{V}{m}\right)$$

where $\alpha z = Np$

$$\Rightarrow \boxed{\alpha = \frac{Np}{m}} \quad \text{(attenuation constant)}$$

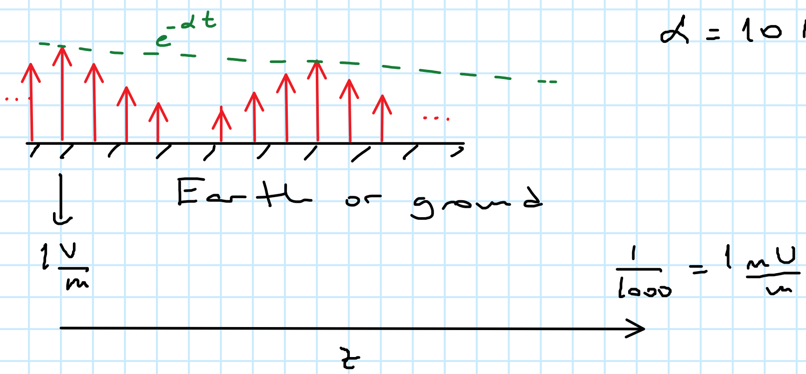
TY - JOUR
AU - Papadopoulos, Theofilos
AU - Papagiannis, Grigoris
AU - Labridis, Dimitris
PY - 2009/03/01
SP - 1064
EP - 1067
T1 - Wave Propagation Characteristics of Overhead Conductors Above Imperfect Stratified Earth for a Wide Frequency Range
VL - 45
DO - 10.1109/TMAG.2009.2012580
JO - Magnetics, IEEE Transactions on ER

Ex:

A vertical polarized dipole antenna radiates ground waves. The attenuation is 10^{-3} Np/m. Find the distance at which the amplitude drops to 1000 of its original value?

Ans:

$\alpha = 10 \ \frac{Np}{m}$



$$\cancel{E_0} e^{-\alpha z} = \frac{\cancel{E_0}}{1000}$$

$$e^{-10^{-3} z} = \frac{1}{1000}$$

$$e^{10^{-3} z} = 1000 \Rightarrow 10^{-3} z = \ln 1000 \Rightarrow z = 6.9 \text{ km.}$$