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– Electromagnetic Waves (Plane Waves) –

We have the following equations from electrostatics:

1-) $\bar{\nabla} \times \bar{E} = 0$
2-) $\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon_0}$
(Electrostatic)

1-) $\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$
2-) $\bar{\nabla} \cdot \bar{B} = 0$
(Magnetostatic)

Re-arranging these equations. (DC only)

1-) $\bar{\nabla} \times \bar{E} = 0$
2-) $\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$
3-) $\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon_0}$
4-) $\bar{\nabla} \cdot \bar{B} = 0$

Differential forms (point forms)

Integral forms

1-) $\oint_C \bar{E} \cdot d\bar{l} = 0$ (KVL)
2-) $\oint_C \bar{B} \cdot d\bar{l} = \mu_0 I$ (Ampere's law.)
3-) $\oint_S \bar{E} \cdot d\bar{s} = \frac{Q}{\epsilon_0}$ (Gauss law)
4-) $\oint_S \bar{B} \cdot d\bar{s} = 0$

Generalized forms: (AC + DC)

1-) $\bar{\nabla} \times \bar{E} = -\frac{d}{dt}\bar{B}$ (Faraday's law) $\longrightarrow$ $\oint_C \bar{E} \cdot d\bar{l} = V_{ind} = \frac{-d}{dt}\phi$

2-) $\bar{\nabla} \times \bar{B} = \mu\left(\bar{J} + \frac{d}{dt}\epsilon \bar{E}\right)$ (Ampere's law) $\longrightarrow$ $\oint_C \bar{B} \cdot d\bar{l} = \mu I + \frac{d}{dt}\mu \oint_S \epsilon \bar{E} \cdot d\bar{s}$

(Maxwell's contribution: $\frac{d}{dt}$ term)

3-) $\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon_0}$

4-) $\bar{\nabla} \cdot \bar{B} = 0$

Integral forms.

These 4 equations are called "Maxwell's equations", and they are all the equations necessary to solve any electromagnetic problem.

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Constitutive Parameters (Electrical properties of a medium)

1-) Permittivity = ϵ = gives the amount of polarization.

If ϵ is high, this means highly polarized medium.

For instance, water is such a medium. The $\bar{E}$-field is decreased inside such mediums

$$\epsilon = \epsilon_0 \epsilon_r \; , \; \epsilon_0 = 8.854 \times 10^{-12} \left(\frac{F}{m}\right), \; \epsilon_r = \text{Relative permittivity}$$

2-) Permeability = μ = gives the amount of magnetization (e^- spin) of a matter. If μ is high, more magnetization when inserted into a magnetic field. Fe is an example of highly magnetized material (Ferrites).

$$\mu = \mu_0 \mu_r \; , \; \mu_0 = 4\pi \times 10^{-7} \left(\frac{H}{m}\right), \; \mu_r = \text{relative permeability.}$$

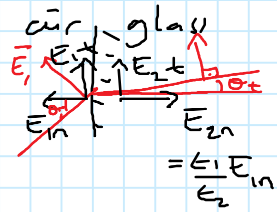
3-) Conductivity = σ = gives how much current flows in the presence of E-field in the medium.

$$\bar{J} = \sigma \bar{E} \quad \text{(ohm's law in points form)}$$

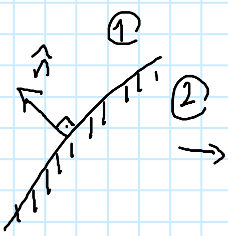
$\bar{J}$: Current density $\left(\frac{A}{m^2}\right)$; $\bar{E}$: Electric field $\left(\frac{V}{m}\right)$

If σ is high like in metals, then small $\bar{E}$-field can cause significant current

Boundary Conditions. Dielectric - Dielectric, Metal - Dielectric, X



$E_{2n} = \frac{\epsilon_1}{\epsilon_2} E_{1n}$



	General	Finite conductivity media, no sources or charges $\sigma_1, \sigma_2 \neq \infty$ $\mathcal{J}_s = 0; q_{es} = 0$ $\mathcal{M}_s = 0; q_{ms} = 0$	Medium 1 of infinite electric conductivity ($\mathscr{E}_1 = \mathscr{H}_1 = 0$) $\sigma_1 = \infty; \sigma_2 \neq \infty$ $\mathcal{M}_s = 0; q_{ms} = 0$	Medium 1 of infinite magnetic conductivity ($\mathscr{E}_1 = \mathscr{H}_1 = 0$) $\mathcal{J}_s = 0; q_{es} = 0$
Tangential electric field intensity	$-\hat{n} \times (\mathscr{E}_2 - \mathscr{E}_1) = \mathcal{M}_s$	$\hat{n} \times (\mathscr{E}_2 - \mathscr{E}_1) = 0$ ↪ $E_{2t} = E_{1t}$	$\hat{n} \times \mathscr{E}_2 = 0$	$-\hat{n} \times \mathscr{E}_2 = \mathcal{M}_s$
Tangential magnetic field intensity	$\hat{n} \times (\mathscr{H}_2 - \mathscr{H}_1) = \mathcal{J}_s$	$\hat{n} \times (\mathscr{H}_2 - \mathscr{H}_1) = 0$	$\hat{n} \times \mathscr{H}_2 = \mathcal{J}_s$	$\hat{n} \times \mathscr{H}_2 = 0$
Normal electric flux density	$\hat{n} \cdot (\mathscr{D}_2 - \mathscr{D}_1) = q_{es}$	$\hat{n} \cdot (\mathscr{D}_2 - \mathscr{D}_1) = 0$ → $D_{1n} = \epsilon_1 E_{1n}$, $\epsilon_2 E_{2n} = \epsilon_1 E_{1n}$	$\hat{n} \cdot \mathscr{D}_2 = q_{es}$	$\hat{n} \cdot \mathscr{D}_2 = 0$
Normal magnetic flux density	$\hat{n} \cdot (\mathscr{B}_2 - \mathscr{B}_1) = q_{ms}$	$\hat{n} \cdot (\mathscr{B}_2 - \mathscr{B}_1) = 0$	$\hat{n} \cdot \mathscr{B}_2 = 0$	$\hat{n} \cdot \mathscr{B}_2 = q_{ms}$

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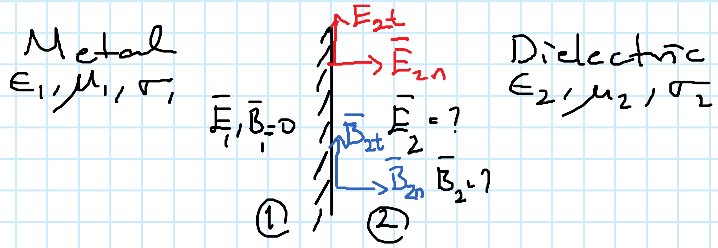
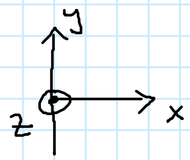
This table gives us the relation between the fields ($\bar{E}$ and $\bar{B}$) at the boundary of two different mediums.

Ex:

Consider a metal to dielectric interface





Let us suppose that $\bar{E}_1$ and $\bar{B}_1$ at the boundary are known ↗ 0 for metal.
By using the table, we can obtain $\bar{E}_2$ and $\bar{B}_2$ at the boundary

Find $\bar{E}_2$ and $\bar{B}_2$

Ans

We can use the 2^{nd} column in the table where medium 1 is the metal and medium 2 is the dielectric:

Inside metals: $\bar{E} = 0 \Rightarrow \bar{E}_{1t} = \bar{E}_{1n} = 0$, $B_{1t} = B_{1n} = 0$

→ Tangential $\bar{E}$-field: $\hat{n} \times \bar{E}_{2t} = 0 \Rightarrow -\hat{a}_x \times \bar{E}_{2t} = 0 \Rightarrow \boxed{\bar{E}_{2t} = 0}$

→ Normal $\bar{E}$-field: $\hat{n} \cdot \epsilon_2 \bar{E}_{2n} = \rho_s \left(\frac{C}{m^2}\right) \Rightarrow -\hat{a}_x \cdot \epsilon_2 \bar{E}_{2n} = \rho_s$

$$\Rightarrow -\hat{a}_x \cdot \epsilon_2 \hat{a}_x E_{2n} = \rho_s$$

$$\Rightarrow \boxed{E_{2n} = \frac{\rho_s}{\epsilon_2}} \left(\frac{V}{m}\right)$$

→ Tangential Mag field: $\hat{n} \times \bar{H}_2 = \bar{J}_s \Rightarrow -\hat{a}_x \times \hat{a}_y \frac{B_{2t}}{\mu_2} = -\hat{a}_z J_s$

$$\Rightarrow \boxed{B_{2t} = \mu_2 J_s \left(\frac{Wb}{m^2}\right)}$$

→ Normal mag. field: $\hat{n} \cdot \bar{B}_2 = 0 \Rightarrow \boxed{B_{2n} = 0}$

—o—

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- In summary, $\bar{E}$ and $\bar{B}$ are zero inside metals.
- ρ_s and $\bar{J}_s = 0$ inside dielectrics and $\sigma = 0$
- for metals, $\sigma = \infty$, ρ_s and $J_s \neq 0$

—o—

- Wave Equation -

Let us re-write Maxwell's equations:

Integral forms:

1-) $\bar{\nabla} \times \bar{E} = -\frac{d}{dt}\bar{B}$ (Faraday's law) $\longrightarrow \oint_C \bar{E} \cdot \bar{dl} = V_{ind} = \frac{-d}{dt}\phi$

2-) $\bar{\nabla} \times \bar{B} = \mu \bar{J} + \frac{d}{dt}(\mu \epsilon \bar{E})$ (Ampere's law) $\longrightarrow \oint_C \bar{B} \cdot \bar{dl} = \mu I + \frac{d}{dt}\mu \oint_S \epsilon \bar{E} \cdot \bar{ds}$

($\frac{d}{dt}(\mu\epsilon\bar{E})$: Maxwell's contribution.)

3-) $\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon}$ $\longrightarrow \oint_S \bar{E} \cdot ds = \frac{Q}{\epsilon}$

4-) $\bar{\nabla} \cdot \bar{B} = 0$ $\longrightarrow \oint_S \bar{B} \cdot \bar{ds} = 0$

We can solve eqn. (1) and (2) simultaneously to get the wave equation (point forms)

Take the curl of 1st eqn:

$$\bar{\nabla} \times \bar{\nabla} \times \bar{E} = -\bar{\nabla} \times \frac{\partial \bar{B}}{\partial t} = -\frac{\partial}{\partial t}(\bar{\nabla} \times \bar{B})$$

Using the vector identity

$$\bar{\nabla} \times \bar{\nabla} \times \bar{A} = \nabla(\bar{\nabla} \cdot \bar{A}) - \nabla^2 A$$

(∇^2 ↳ Laplacian operator (2nd order space derivatives))

Then, we get

$$\bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}(\bar{\nabla} \times \bar{B})$$

$$\bar{\nabla} \times \bar{B} = \mu \bar{J} + \frac{\partial}{\partial t}\mu\epsilon\bar{E} \quad (2^{nd} \text{ eqn.})$$

Then,

$$\bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}\left(\mu \bar{J} + \frac{\partial}{\partial t}\mu\epsilon\bar{E}\right)$$

or

$$\underbrace{\bar{\nabla}(\bar{\nabla} \cdot \bar{E})}_{\frac{\rho}{\epsilon_0}} - \nabla^2 \bar{E} = -\frac{\partial}{\partial t}\mu\bar{J} - \frac{\partial^2}{\partial t^2}\mu\epsilon\bar{E}$$

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$$\bar{\nabla}(\bar{\nabla}\cdot\bar{E}) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}\mu\bar{J} - \frac{\partial^2}{\partial t^2}\mu\epsilon\bar{E}$$

$\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon}$ Also, $\bar{J} = \sigma\bar{E}$

Then, we have

$$\frac{1}{\epsilon}(\bar{\nabla}\cdot\rho) - \nabla^2\bar{E} = -\frac{\partial}{\partial t}(\mu\sigma\bar{E}) - \frac{\partial^2}{\partial t^2}(\mu\epsilon\bar{E})$$

Re-arranging the terms:

$$\boxed{\nabla^2\bar{E} = \frac{1}{\epsilon}(\bar{\nabla}\cdot\rho) + \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}}$$ (Wave equation for $\bar{E}$-field)

We could do the same analysis for $\bar{H}$, we would have

$$\boxed{\nabla^2\bar{H} = \frac{1}{\mu}(\bar{\nabla}\cdot\rho_m) + \mu\sigma\frac{\partial\bar{H}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{H}}{\partial t^2}}$$ (Wave equ. for $\bar{H}$-field)

$\rho_m = 0$ → magnetic charge density $= 0$ (fictitious) $\bar{B} = \mu\bar{H}$

Solution of these equations:

Let us use the wave eqn. for $\bar{E}$-field:

$$\nabla^2\bar{E} = \frac{1}{\epsilon}(\bar{\nabla}\cdot\rho) + \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2}$$

We want a solution in dielectric medium (air)

Air

ρ $\bar{J}$ $\times$ $\bar{E}, \bar{H}$ $(\bar{E}(x,y,z), \bar{H}(x,y,z)) \longrightarrow z$

In air (dielectric), $\sigma = 0$, $\rho = J = 0$ — Simple media

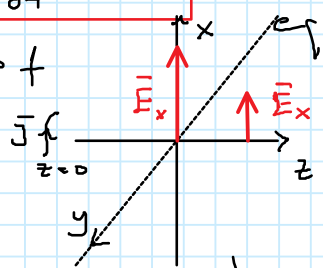
Wave Solution for Simple Medium (Source free and lossless) $\bar{J} = \rho = 0$, $\sigma = 0$

Then we obtain:

$$\boxed{\nabla^2\bar{E} - \mu\epsilon\frac{\partial^2\bar{E}}{\partial t^2} = 0}$$ (Homogenous wave equation for simple medium)

Assume $\boxed{\bar{E} = \hat{a}_x E_x(z,t)}$ type of solution. (plane wave)

$$\Rightarrow \nabla^2\bar{E} = \hat{a}_x\frac{\partial^2 E_x(z,t)}{\partial z^2}$$



Substitute this into the wave equation

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$$\hat{a}_x \frac{\partial^2 E_x(z,t)}{\partial z^2} - \mu \epsilon \, \hat{a}_x \frac{\partial^2 E_x(z,t)}{\partial t^2} = 0$$

Initial conditions = Boundary conditions:

1-) $E_x(0) = E_0$

2-) $E_x\left(\frac{2\pi}{k}\right) = E_0$ (periodic) $k = \omega\sqrt{\mu\epsilon}$, ω = radian frequency.

The solution of this linear 2nd order homogenous differential equation is:

$$E_x(z,t) = E_0 \cos(\omega t - kz) \quad \text{(wave solution)}$$

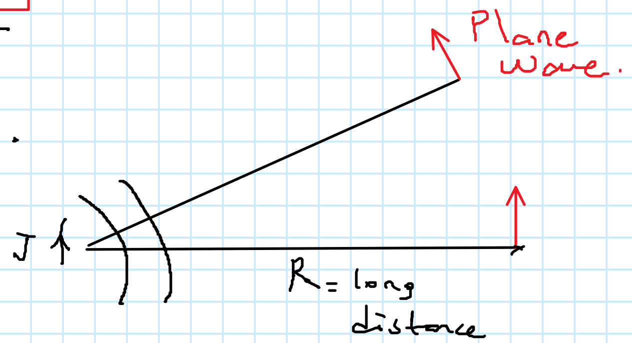
, where k = wave number

2 important remarks:

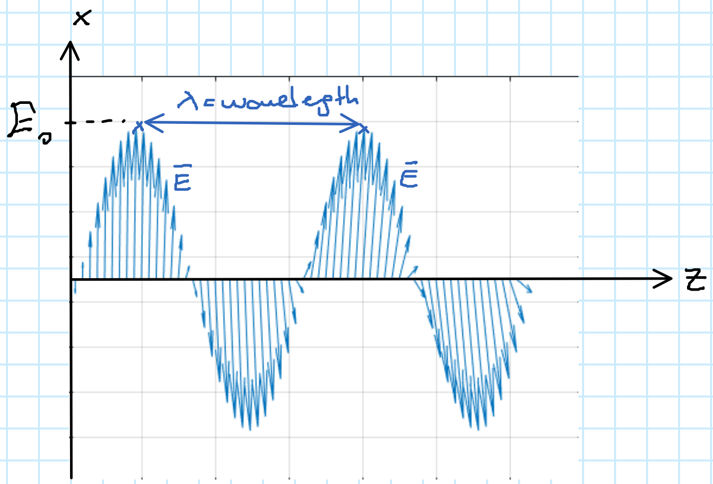
1-) At its origin, $\bar{J}$ causes $\bar{E}$ and $\bar{H}$.

2-) Assumption on the solution

$$\bar{E} = \hat{a}_x E_x(z,t)$$



Solution:



Phase Velocity:

Consider the plane wave solution

$$E_x(z,t) = E_0 \cos(\omega t - kz) \quad \left(\frac{V}{m}\right)$$

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To find phase velocity

$$\omega t - kz = C_0 = \text{constant}$$

Take the derivative of both sides wrt. t.

$$\Rightarrow \frac{d}{dt}(\omega t) - k\frac{dz}{dt} = \frac{d}{dt}C_0$$

$$\omega - k . v_p = 0$$

$$\Rightarrow \boxed{v_p = \frac{\omega}{k} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu\epsilon}}}$$

For air: $v_p = c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = 3\times10^8$ m/s. (speed of light)

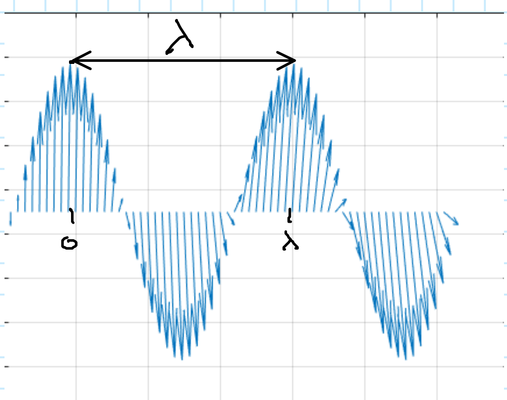
$$v_p\Big|_{water} = \frac{1}{\sqrt{\mu_0\epsilon_r\epsilon_0}} = \frac{1}{9\sqrt{\mu_0\epsilon_0}} = \frac{3}{9}\times10^8 \text{ m/s}.$$

($\epsilon_r \to 81$)

Wavelength:

Consider the plane wave solution

$$E_x(z,t) = E_0\cos(\omega t - kz) \quad \left(\frac{V}{m}\right)$$



At $z=0$: $(t=0)$

$\omega t - kz = 0$

$0 = kz$

$\Rightarrow z = 0$

At $z = \lambda$: $(t = T)$

$\omega t = k\lambda$

$\omega T = k\lambda$

$$\Rightarrow \lambda = \frac{\frac{2\pi}{T}\cdot T}{k}$$

λ = Distance between a cycle of a wave.

$$\boxed{\lambda = \frac{2\pi}{k} \text{ (m.)}}$$

or since $k = \omega\sqrt{\mu\epsilon}$ $\Rightarrow$ $\lambda = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{2\pi}{2\pi f\sqrt{\mu\epsilon}} = \frac{1}{f\sqrt{\mu\epsilon}} = \frac{v_p}{f}$

$$\Rightarrow \lambda = \frac{v_p}{f}$$

Ex:

Find the wavelength of a wave at 2.4 GHz (Wi-Fi) in air?

Ans $\lambda = \frac{3\times10^8 \text{ m/s}}{2.4\times10^9 \frac{1}{s}} = \frac{3}{2.4}\times10^{-1} = 0.125$ m $\Rightarrow \lambda = 125$ mm.

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- General plane wave solution has two terms:

$$\bar{E}(z,t) = \underbrace{E_0 \cos(\omega t - kz)}_{\text{Incident wave}} + \underbrace{E_0 \cos(\omega t + kz)}_{\text{Reflected wave}}$$

Wave Solution in Source Free and Lossy Medium: ($\bar{J} = \rho = 0$, $\sigma \neq 0$)

Ex: water, concrete.

The general wave equation is given as

$$\nabla^2 \bar{E} = \frac{1}{\epsilon}(\nabla \cdot \rho) + \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

For $J = \rho = 0$ (source free) medium, we get

$$\nabla^2 \bar{E} = \mu\sigma \frac{\partial \bar{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

The solution of this equation is: (Assuming a plane wave)

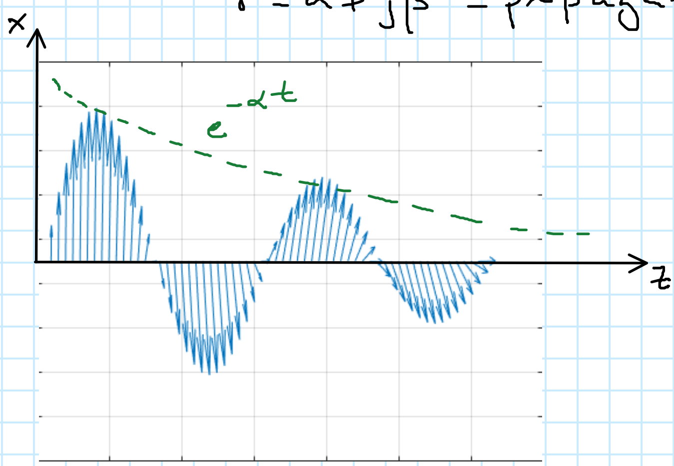
$$\bar{E}(z,t) = E_0 e^{-\alpha t} \cos(\omega t - \beta z)$$

where α = attenuation constant

β = phase constant

and

$\gamma = \alpha + j\beta$ = propagation constant



- For metals, σ is very high, so is α. Thus, wave attenuates very quickly. In fact, only surface charge remains

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- The relationship between α, β and σ and other constants are given in the table below (ω = radian frequency $= 2\pi f$)

E.M. Wave Propagation properties	Exact	Good dielectric $\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \ll 1$	Good conductor $\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \gg 1$
Attenuation constant α	$= \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} - 1\right]\right\}^{1/2}$	$\simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
Phase constant β	$= \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2} + 1\right]\right\}^{1/2}$	$\simeq \omega\sqrt{\mu\varepsilon}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
Wave Z_w intrinsic η_c impedances $Z_w = \eta_c$	$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$	$\simeq \sqrt{\frac{\mu}{\varepsilon}}$	$\simeq \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$
Wavelength λ	$= \frac{2\pi}{\beta}$	$\simeq \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$	$\simeq 2\pi\sqrt{\frac{2}{\omega\mu\sigma}}$
Velocity v	$= \frac{\omega}{\beta}$	$\simeq \frac{1}{\sqrt{\mu\varepsilon}}$	$\simeq \sqrt{\frac{2\omega}{\mu\sigma}}$
Skin depth δ	$= \frac{1}{\alpha}$	$\simeq \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$	$\simeq \sqrt{\frac{2}{\omega\mu\sigma}}$

Ex: X

For a wave at 2.4 GHz (WiFi/WiMax) find the wave velocity, wavelength, attenuation constant and phase constant for

a-) in air b-) in sea water c-) in concrete.

Given that:

$\epsilon_0 = 8.854 \times 10^{-12}$, $\mu_0 = 4\pi \times 10^{-7}$, $\epsilon_r\big|_{\text{sea water}} = 70$, $\epsilon_r\big|_{\text{concrete}} = 10$

and

$\sigma_{air} = 0$, $\sigma_{\text{sea water}} = 5\ \frac{S}{m}$, $\sigma_{\text{concrete}} = \frac{1}{100\ \Omega m} = 10^{-2}\ \frac{S}{m}$

Ans

a-) In air: $\omega = 2\pi f = 2\pi(2.4 \times 10^9) = 1.5 \times 10^{10} = 15 \times 10^9$ rad/s

We check the condition for $\frac{\sigma}{\omega\epsilon} = 0 \Rightarrow \alpha = 0$.

and $\beta = \omega\sqrt{\mu\epsilon} = 15\times10^9\sqrt{\mu_0\epsilon_0} = 15\times10^9\sqrt{(4\pi\times10^{-7})(8.854\times10^{-12})} = 50$

Velocity $= \frac{\omega}{\beta} = \frac{15\times10^9}{50} = 3\times10^8$ m/s, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{50} = \frac{\pi}{25} = 0.125\ m = 12.5\ cm$

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b-) For water: $\frac{\sigma}{\omega\epsilon} = \frac{5}{(15\times10^9)(70\times 8.854\times10^{-12})} = 0.5378$

$\epsilon \to \epsilon_0\epsilon_r$

$\to \frac{1}{10}$ th of 1 is considered as "≪".

and $\left(\frac{\sigma}{\omega\epsilon}\right)^2 = 0.2 \Rightarrow$ $0.2 \ll 1$. Thus, we can use good dielectric approximation.

$$\Rightarrow \alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{5}{2}\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} = \frac{5}{2}\cdot\frac{1}{\sqrt{\epsilon_r}}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{5\cdot 377}{2}\cdot\frac{1}{\sqrt{70}}$$

($\mu_0 = 4\pi\times10^{-7}$, $\epsilon_0 = 8.854\times10^{-12}$, $\sqrt{\mu_0/\epsilon_0} = 377$)

$$\alpha = 112.65$$

and $\beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\cdot\sqrt{\epsilon_r} = 50\cdot\sqrt{70} = 418.$

velocity $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_0}}\cdot\frac{1}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{\sqrt{70}} = 3.58\times10^7$ m/s.

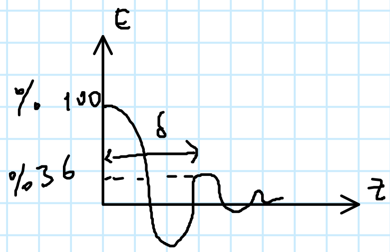
($\epsilon \to \epsilon_0\epsilon_r$, $\frac{1}{\sqrt{\mu_0\epsilon_0}} = c$)

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{418} = 1.5\text{cm}.$$

c-) $\left(\frac{\sigma}{\omega\epsilon}\right)^2 = \frac{10^{-2}}{(15\times10^9)(70\cdot 8.854\times10^{-12})} = 1.15\times10^{-6} \ll 1$

⋮

HW#4: Complete this example.



Skin Depth:

For lossy mediums, the 'skin depth' is the distance for the electromagnetic wave to lose its amplitude by $\frac{1}{e}$, where $e = 2.71828\ldots$

$\Rightarrow$ Define skin depth as:

$$\delta = \frac{1}{\alpha} \quad (\text{m})$$

The proof:

The plane wave expression of an e.m wave in lossy medium is

$$\bar{E}(z,t) = \hat{a}_x E_0 e^{-\alpha z}\cos(\omega t - \beta z) \quad \text{(Incident wave)}$$

E_0 (max for when $z=0$).

For $z = \delta = \frac{1}{\alpha}$

$$\bar{E}(z=\delta, t) = \hat{a}_x \underbrace{E_0 e^{-\alpha\frac{1}{\alpha}}}_{E_0\cdot\frac{1}{e}}\cos(\omega t - \beta z)$$

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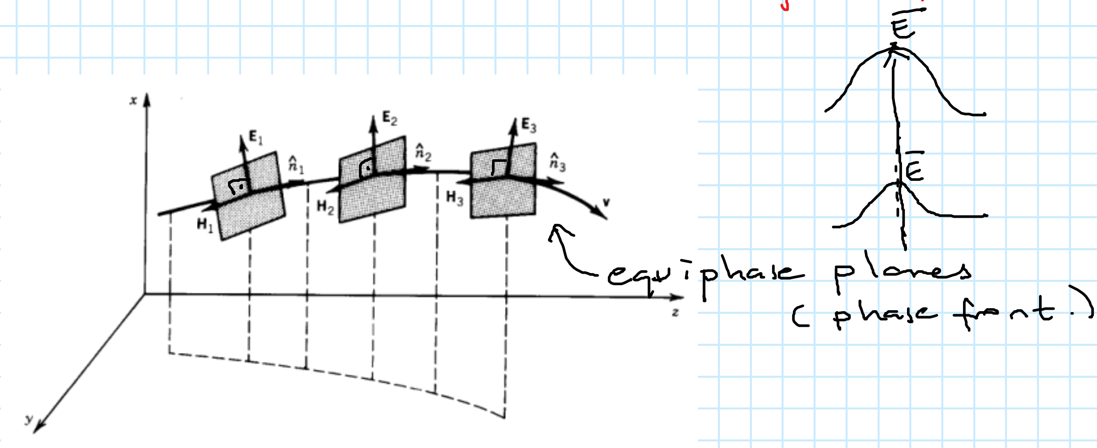
– Wave Propagation and Plane Waves –

Electromagnetic wave modes are defined as being particular field configurations. They are:

1-) TEM mode = (Transverse Electric and Magnetic)



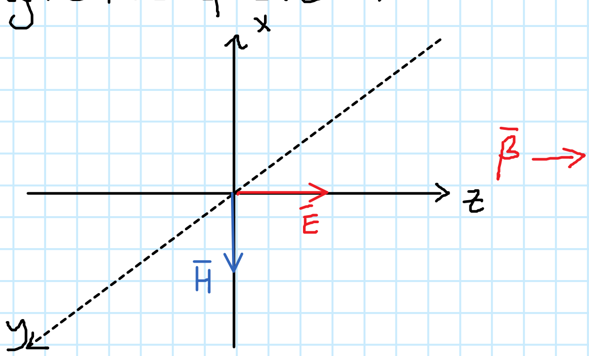
TEM: Transverse Electric and Magnetic.

Electric and Magnetic fields are perpendicular to the direction of propagation.

2-) TM Wave - (Transverse Magnetic)

There is no magnetic field in the direction of propagation.

TM^z: ← direction of prop.

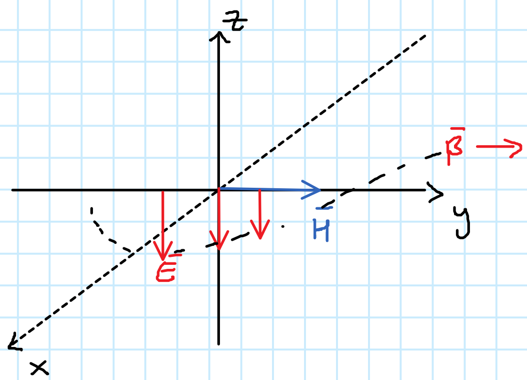


Equiphase plane = xy plane.

3-) TE Wave : (Transverse Electric)

There is no $\bar{E}$-field in the direction of propagation.

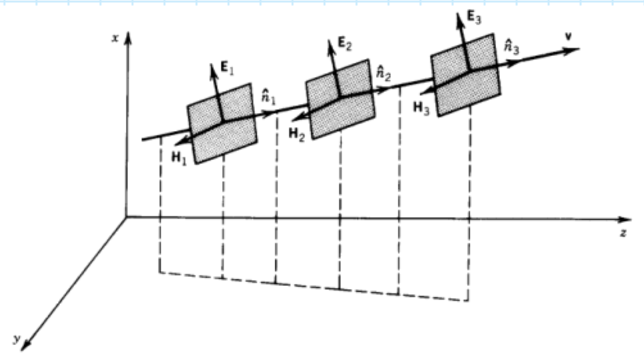
TE^y:



Plane Wave:

Electric and magnetic fields are contained in a local plane on which waves have equal phase (equiphase plane).

A plane wave is a TEM wave with planar equiphase surfaces.



Plane Wave

Uniform Plane Wave: Wave amplitudes are uniform over the equiphase plane.

In TEM mode, no $\bar{E}$ or $\bar{H}$ are on the direction of propagation. $\bar{E}$ and $\bar{H}$ are on the equiphase plane

- All waves converge to a planar at far away from the source.

—o—

- Plane Wave Equations and Time Harmonic Fields -

$\bar{E}(z,t) = \hat{a}_x E_o \cos(\omega t - kz)$, k = wave number

- Since Maxwell's equations and the wave equations are linear, and they are in sinusoidal form, we can use "phasors" to simplify our calculations

$\Rightarrow \bar{E}(z) = \hat{a}_x E_o e^{-jkz}$ (Electric field wave phasor expression.) ⇓ Time Harmonic

Similarly for magnetic field:

$\Rightarrow \bar{H}(z) = \hat{a}_y H_o e^{-jkz}$ (for plane wave)

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- For plane waves, the general expression in lossless media is:

$$\bar{E}(\bar{R}) = \hat{a}_E E_o e^{-j\bar{k}\cdot\bar{R}}$$

, $\hat{a}_E$ = Unit vector in the direction of $\bar{E}$

where $\bar{R}$ = Position vector from origin to a point on equiphase plane $= \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

$\bar{k}$ = Wavenumber vector $= \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z$

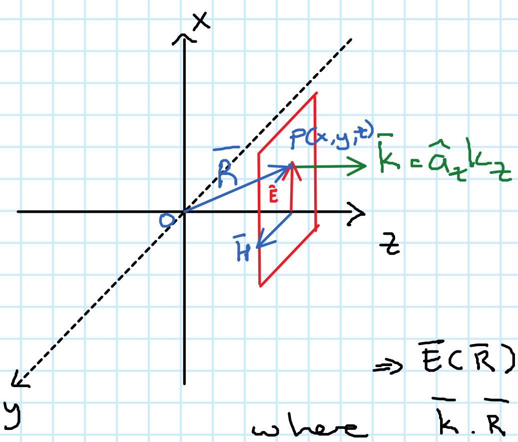
<u>Ex:</u>

Use the general expression for a plane wave electric field which is x directed and moving towards z direction

<u>Ans:</u>

$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z$ ($k_x \to 0$, $k_y \to 0$)

(Wavenumber vector is in the direction of propagation)



$\bar{R} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

$\Rightarrow \bar{E}(\bar{R}) = \hat{a}_x E_o e^{-j\bar{k}\cdot\bar{R}}$

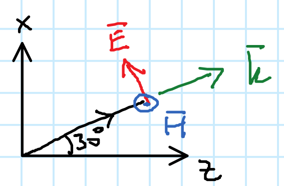
where $\bar{k}\cdot\bar{R} = (\hat{a}_z k_z)\cdot(\bar{R}) = (\hat{a}_z k_z)\cdot(\hat{a}_x x + \hat{a}_y y + \hat{a}_z z) = k_z z$

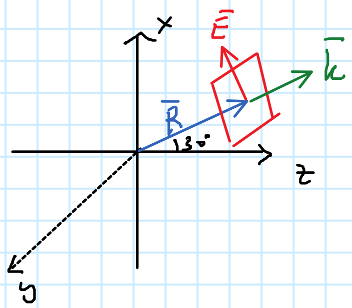
$\Rightarrow \bar{E}(\bar{R}) = \hat{a}_x E_o e^{-j k_z z}$ or

$\bar{E}(z) = \hat{a}_x E_o e^{-j k_z z}$ (Phasor)

<u>Ex.</u> X

Find the electric field expression for a plane wave whose direction of propagation is 30° with the z-axis:





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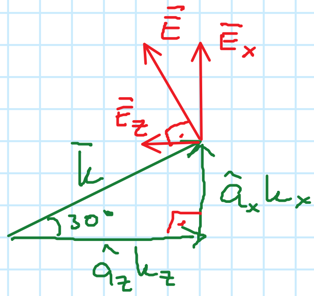
Ans:



Using $\bar{E}(\bar{R}) = \hat{a}_z E_o e^{-j\bar{k}\cdot\bar{R}}$

where

$$\vec{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z = \hat{a}_x k_x + \hat{a}_z k_z$$

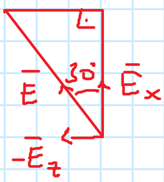
($k_y = 0$)

$$= \hat{a}_x k \sin 30 + \hat{a}_z k \cos 30°$$

$$= \hat{a}_x \frac{k}{2} + \hat{a}_z \frac{\sqrt{3}\,k}{2}$$

$\bar{R} = \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$.

$\Rightarrow \bar{k}\cdot\bar{R} = x\frac{k}{2} + z\frac{k\sqrt{3}}{2}$



Also, $\bar{E} = \hat{a}_x E_x - \hat{a}_z E_z$

$\bar{E}_x$: $E_o \cdot \cos 30$ $\bar{E}_z$: $E_o \cdot \sin 30$

$$\Rightarrow \boxed{\bar{E}(\bar{R}) = E_o\left(\hat{a}_x \frac{\sqrt{3}}{2} - \frac{\hat{a}_z}{2}\right) e^{-jk\left(\frac{x}{2} + \frac{z\sqrt{3}}{2}\right)} \quad \left(\frac{V}{m}\right)} \quad \text{(Phasor)}$$

- For a plane wave:

$$\boxed{\bar{H}(\bar{R}) = \frac{1}{\eta}\,\hat{a}_n \times \bar{E}(\bar{R}) \quad \left(\frac{A}{m}\right)}$$

where $\hat{a}_n$ = Unit vector in the direction of propagation.

and η = Point impedance or intrinsic impedance.

and $\eta = Z_w = \sqrt{\frac{\mu}{\epsilon}}$ (for dielectrics)

For air $\eta = \sqrt{\frac{\mu_o}{\epsilon_o}} = \sqrt{\frac{4\pi\times 10^{-7}}{8.854\times 10^{-12}}} = 377\,\Omega$.

For example, in the previous question if we wanted to find $\bar{H}$:

$$\hat{a}_n = \frac{\bar{k}}{\|\bar{k}\|} = \frac{\hat{a}_x \frac{k}{2} + \hat{a}_z \frac{\sqrt{3}\,k}{2}}{\sqrt{\frac{k^2}{4} + \frac{3}{4}k^2}}, \quad k = \omega\sqrt{\mu\epsilon} = \frac{\omega}{c}$$

⋮

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Wave Power:

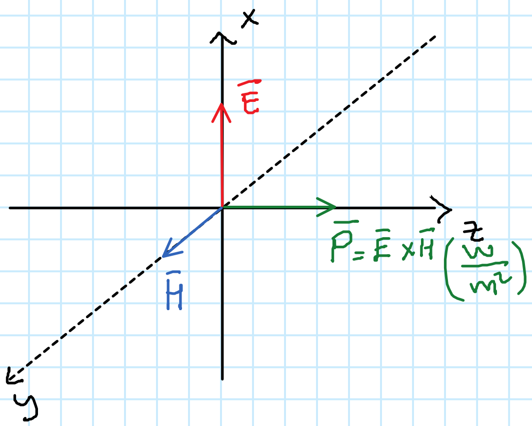
Instantaneous wave power density: $\bar{P} = \bar{E} \times \bar{H} \; \left(\frac{W}{m^2}\right)$ (Poynting vector)

Total Wave power:

$$P_{total} = \int_{sphere} \bar{P} \cdot d\bar{s} \quad (W)$$

Average Power Density: $\bar{P}_{avg} = \frac{1}{2}(\bar{E} \times \bar{H}^*)$ (W/m^2)

$\bar{P}_{avg} = \bar{E}_{eff} \times \bar{H}^*_{eff}$ (W/m^2)



—o—

Ex: X

Find the electric and magnetic field plane waves for (η=377Ω). Also, find $\bar{P}$ and $\bar{P}_{avg}$.

$$\bar{E} = \hat{a}_x E_o e^{-jkz} \quad \text{(incident wave)}.$$

Ans.

$\bar{k} = \hat{a}_z k_z$ (propagation is in the z-direction.)

$\hat{a}_n = \hat{a}_z$

$$\Rightarrow \quad \bar{H}(\bar{R}) = \frac{1}{\eta} \hat{a}_n \times \bar{E}(\bar{R})$$

$$= \frac{1}{377} \hat{a}_z \times \hat{a}_x E_o e^{-jkz}$$

$$= \frac{1}{377} \hat{a}_y E_o e^{-jkz} = \hat{a}_y \frac{E_o}{377} e^{-jkz}$$

The instantaneous power density vector:

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$$\bar{P} = \bar{E} \times \bar{H} = \hat{a}_x E_o e^{-jkz} \times \hat{a}_y \frac{E_o}{\eta} e^{-jkz} = \hat{a}_z \frac{E_o^2}{\eta} e^{-2jkz} \left(\frac{W}{m^2}\right)$$

Note that

$$Re[(\bar{E} \times \bar{H}) e^{j\omega t}] \neq Re[\bar{E} . e^{j\omega t}] \times Re[\bar{H} e^{j\omega t}]$$

Because the evaluation of power is non-linear.
(E is multiplied by H).

$$\Rightarrow P(z,t) = Re[\bar{E} . e^{j\omega t}] \times Re[\bar{H} . e^{j\omega t}]$$

$$= \hat{a}_x E_o \cos(\omega t - kz) \times \hat{a}_y \frac{E_o}{\eta} \cos(\omega t - kz)$$

$$= \hat{a}_z \frac{E_o^2}{\eta} \cos^2(\omega t - kz)$$

$$= \hat{a}_z \frac{E_o^2}{\eta} \cos(2\omega t - 2kz) \left(\frac{W}{m^2}\right)$$ (Time dependent expression)

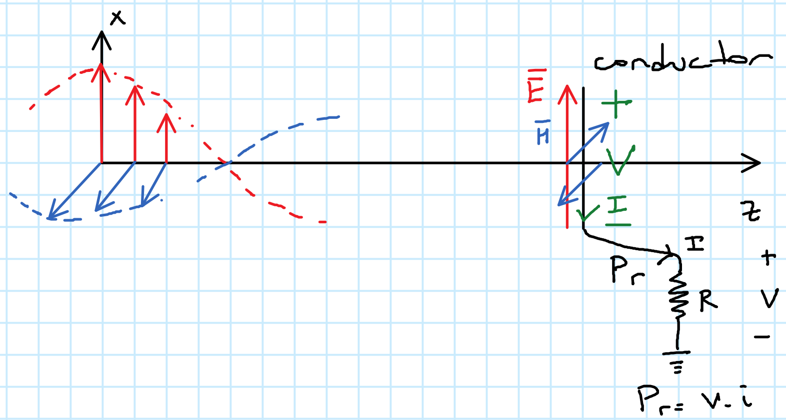
$$P_{avg} = \frac{1}{2} Re[\bar{E} \times \bar{H}^*] = \frac{E_o^2}{2\eta} \left(\frac{W}{m^2}\right)$$

In case of a lossy media-

$$P(z,t) = \hat{a}_z \frac{E_o^2}{\eta} e^{-2\alpha z} \cos(2\omega t - 2kz) \left(\frac{W}{m^2}\right)$$

—o—

- There is a relation between fields and circuit quantities:

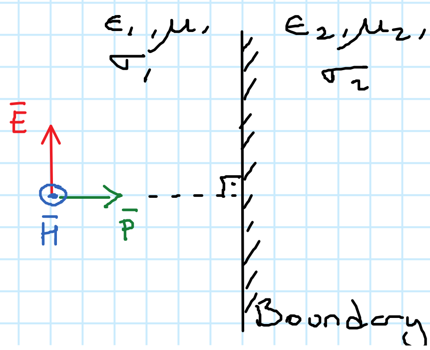


$V = E.d$

$2\pi r H = I$

– Reflection and Transmission of Waves –

1-) Normal Interface: (Lossless case)



Boundary

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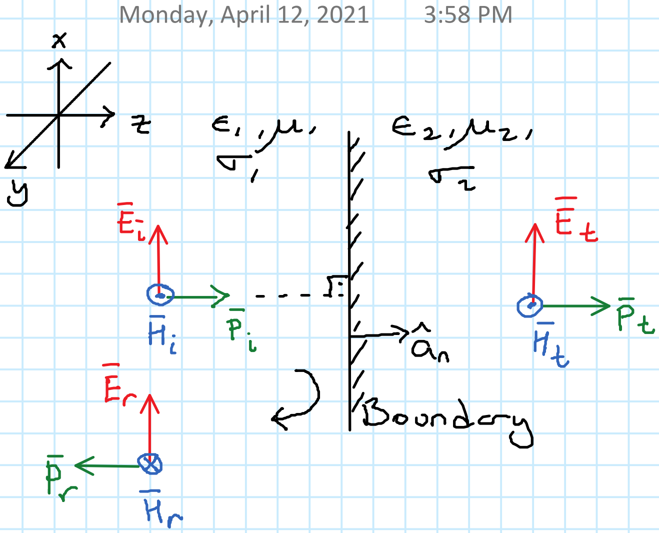

$P_r + P_t = P_i$ (conservation of power)

Analysis:

$\bar{E}_i = \hat{a}_x E_o e^{-jk_1 z}$, incident wave in lossless medium.

$\bar{E}_r = \hat{a}_x \underbrace{\Gamma E_o}_{E_r} e^{+jk_1 z}$, reflected wave where k_1 is the wavenumber of the 1st medium.

where

$$\Gamma = \text{Reflection coefficient} = \frac{E_r}{E_i}$$

If $\Gamma = 0 \Rightarrow E_r = 0$ (Total transmission)

If $\Gamma = 1 \Rightarrow E_r = E_i$ (Total reflection)

$\Rightarrow |\Gamma| = \frac{|\bar{E}_r|}{E_o}$, $0 \leq |\Gamma| \leq 1$

Similarly,

$\bar{E}_t = \hat{a}_x \underbrace{T E_o}_{E_t} e^{-jk_2 z}$, transmitted wave in the 2nd medium.

$$\Rightarrow T = \text{Transmission coefficient} = \frac{E_t}{E_i}$$, $0 \leq |T| \leq 1$

Also,

$$\bar{H}_i = \hat{a}_y \frac{E_o}{\eta_1} e^{-jk_1 z}$$

$$\bar{H}_r = -\hat{a}_y \Gamma \frac{E_o}{\eta_1} e^{+jk_1 z}$$

$$\bar{H}_t = \hat{a}_y T \frac{E_o}{\eta_2} e^{-jk_2 z}$$

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- Imposing the B.C's:

$$|E_1^t| = |E_2^t| \Rightarrow |\bar{E}_i| + |\bar{E}_r| = |\bar{E}_t|$$

$$E_o + \Gamma E_o = T E_o$$

$$\boxed{1 + \Gamma = T} \quad (1)$$

- Imposing another B.C's:

$$|\bar{H}_1^t| = |\bar{H}_2^t| \Rightarrow \frac{E_o}{\eta_1} - \frac{\Gamma E_o}{\eta_1} = \frac{T E_o}{\eta_2}$$

$$\Rightarrow \boxed{\frac{1}{\eta_1}(1-\Gamma) = \frac{1}{\eta_2} T} \quad (2)$$

Solving (1) and (2) simultaneously,

$$\Rightarrow \boxed{\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}} \quad \text{and} \quad \boxed{T = \frac{2\eta_2}{\eta_1 + \eta_2}}$$

- Thus, we can obtain Γ and T, if we know the wave impedances of the mediums

Power Considerations:

$$\bar{P}_{i_{avg}} = \frac{1}{2} Re(\bar{E}_i \times \bar{H}_i^*) = \boxed{\hat{a}_z \frac{E_o^2}{2\eta_1}} \left(\frac{W}{m^2}\right)$$

$$\bar{P}_{r_{avg}} = \frac{1}{2} Re[\bar{E}_r \times \bar{H}_r^*] = \boxed{-\hat{a}_z |\Gamma|^2 |P_{i_{avg}}| \quad \left(\frac{W}{m^2}\right)}$$

$$\bar{P}_{t_{avg}} = \frac{1}{2} Re[\bar{E}_t \times \bar{H}_t^*] = \hat{a}_z |T|^2 \frac{\eta_1}{\eta_2} \cdot |\bar{P}_{i_{avg}}|$$

$$= \boxed{\hat{a}_z [1 - |\Gamma|^2] |\bar{P}_{i_{avg}}|}$$

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Ex:

A uniform plane wave in air is incident normally upon a flat lossless medium with $\epsilon_r = 2.56$ (polystrene). Determine the reflection and transmission coefficients and the power densities in each medium. Assume that the amplitude of the incident E-field is 1mV/m

Ans:

$E_o = |\bar{E}_i| = 1 \frac{mV}{m}$.

Wave impedances. $Z = \sqrt{\frac{\mu}{\epsilon}} = \eta$.

$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_o}{\epsilon_o}}$ ← $\mu_o = 4\pi \times 10^{-7}$, $\epsilon_o = 8.854 \times 10^{-12}$.

and

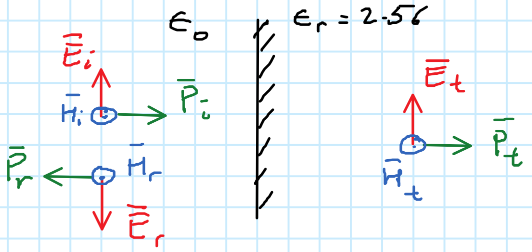
$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_o}{2.56\epsilon_o}} = \frac{1}{1.6}\sqrt{\frac{\mu_o}{\epsilon_o}} = \frac{1}{1.6}\eta_1$$

Thus, Reflection & Transmission coefficients are

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{1.6}\eta_1 - \eta_1}{\frac{1}{1.6}\eta_1 + \eta_1} = \frac{\frac{1}{1.6} - 1}{\frac{1}{1.6} + 1} = \frac{1 - 1.6}{1 + 1.6} = -0.231$$

$$\Rightarrow T = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\left(\frac{1}{1.6}\right)}{1 + \frac{1}{1.6}} = \frac{2}{2.6} = 0.769$$

$\Rightarrow 1 + \Gamma = T \quad \Rightarrow 1 - 0.231 = 0.769$ ✓ from eqn (1)



Power Calculations,

$$|P^i_{avg}| = \frac{E_o^2}{2\eta_1} = \frac{(10^{-3})^2}{2(377)} = 1.327 \times 10^{-9} = 1.327 \ \frac{nW}{m^2}$$

$$|P^r_{avg}| = |\Gamma|^2 \, |\bar{P}^i_{avg}| = |-0.231|^2 (1.327 \times 10^{-9}) = 0.071 \ \frac{nW}{m^2}$$

$$|P^t_{avg}| = (1 - |\Gamma|^2)|\bar{P}^i_{avg}| = |T|^2 \frac{\eta_1}{\eta_2} |P^i_{avg}| = [1 - (0.231)^2] \cdot (1.327 \times 10^{-9}) = 1.256 \ \frac{nW}{m^2}$$

$\Rightarrow |P^i_{avg}| = |P^r_{avg}| + |P^t_{avg}|$

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Ex

Repeat the previous example for air to water interface.

$\epsilon_r\big|_{water} = 81$, $\mu_r\big|_{water} = 1 \Rightarrow \mu_{water} = \mu_0$.

Ans:

$E_0 = |\bar{E}_i| = 1\ \frac{mV}{m}$.

Wave impedances. $Z = \sqrt{\frac{\mu}{\epsilon}} = \eta$.

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_0}{\epsilon_0}} \quad \begin{matrix} \leftarrow 4\pi\times10^{-7} \\ \leftarrow 8.854\times10^{-12} \end{matrix} = 377\,\Omega \text{ (wave impedance of air)}$$

and

$$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_0}{81\epsilon_0}} = \frac{1}{9}\sqrt{\frac{\mu_0}{\epsilon_0}} = \frac{1}{9}\eta_1$$

Thus, Reflection & Transmission coefficients are

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{1}{9}\eta_1 - \eta_1}{\frac{1}{9}\eta_1 + \eta_1} = \frac{\frac{1}{9}-1}{\frac{1}{9}+1} = \frac{1-9}{1+9} = -\frac{8}{10} = -0.8$$

$$\Rightarrow T = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2\left(\frac{1}{9}\right)}{1+\frac{1}{9}} = 0.2$$

Thus, $|\bar{E}_r| = |\bar{E}_i||\Gamma| = (1mV)(0.8) = 0.8\,mV$

$|\bar{E}_t| = |\bar{E}_i||T| = 0.2\,mV$

$$|P^i_{avg}| = \frac{E_0^2}{2\eta_1} = \frac{(10^{-3})^2}{2(377)} = 1.327\times10^{-9} = 1.327\ \frac{nW}{m^2}$$

$$|P^r_{avg}| = |\Gamma|^2\,|\bar{P}^i_{avg}| = (-0.8)^2\ (1.327\times10^{-9}) = 8.5\times10^{-10} = 0.85\frac{nW}{m^2}$$

$$|P^t_{avg}| = (1-|\Gamma|^2)|\bar{P}^i_{avg}| = |T|^2\frac{\eta_1}{\eta_2}\,|P^i_{avg}| = \left[1-(0.8)^2\right]\cdot(1.327\times10^{-9})$$

$$\Rightarrow |P^i_{avg}| = |P^r_{avg}| + |P^t_{avg}| \checkmark \qquad = 0.47\ \frac{nW}{m^2}$$

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Ex:

For the same electric field plane wave with $E_0 = 1\frac{mV}{m}$ consider air to aluminum interface. Find Γ and $T = ?$

For Al $\rightarrow$ $\sigma = 3.54\times10^7 \frac{S}{m}$, $f = 1$ GHz (GSM phone comm.)
$\epsilon_r = 10$.

Ans:

$\eta_1 = 377\,\Omega$ (air), $\eta_2 = ?$ $\Rightarrow \left(\frac{\sigma}{\omega\epsilon}\right)^2 = \left(\frac{3.54\times10^7}{2\pi\times10^9\times10\times8.854\times10^{-12}}\right)^2 \gg 1$

$$\Rightarrow \eta_2 = \sqrt{\frac{\omega\mu}{2\sigma}}(1+j) = \sqrt{\frac{2\pi\times10^9\times4\pi\times10^{-7}}{2(3.54\times10^7)}}(1+j)$$

$$= 0.015e^{j0.7854}$$

$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{0.015e^{j0.78} - 377}{0.015e^{j0.78} + 377} = 0.9999e^{j3.14\,rad}$$

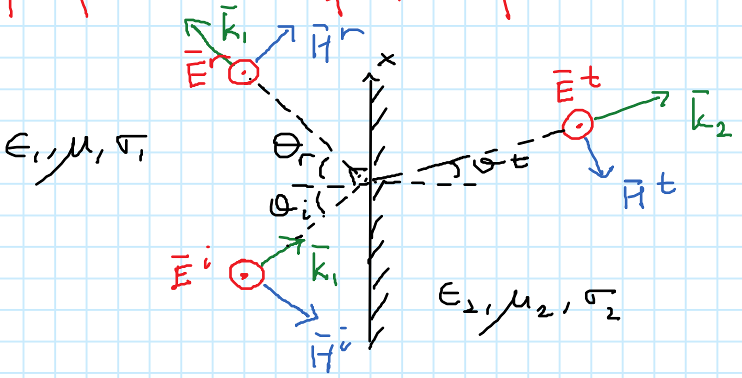
$$= 0.9999e^{j180°}.$$

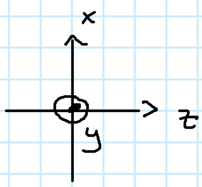
$$\bar{E}_r = \bar{E}_i \cdot \Gamma = \hat{a}_x E_0 e^{-jkz} \cdot \underbrace{0.9999e^{j180°}}_{-1}$$

$$= -\hat{a}_x E_0 e^{jkz}$$

2-) Angled Incidence (Oblique Incident) (Lossless Media).

a-) Perpendicular (TE mode) Polarization: shows the directional properties of the fields.





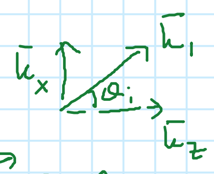
where θ_i = Angle of incidence

θ_r = Angle of reflection

θ_t = Angle of transmission

and

$$\bar{E}_i = \hat{a}_y E_0 e^{-j\,k_1(x\sin\theta_i + z\cos\theta_i)}$$



$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y$

$\bar{k}\cdot\bar{R}$, $\bar{R} \to \hat{a}_x x + \hat{a}_y y + \hat{a}_z z$

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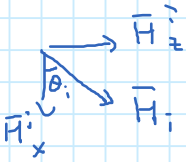

$H^i_x = \hat{a}_x \frac{E_0}{\eta} \cos\theta_i$

$\bar{H}^i_z = \hat{a}_z \frac{E_0}{\eta} \sin\theta_i$.

$$\bar{H}^i = (-\hat{a}_x \cos\theta_i + \hat{a}_z \sin\theta_i)\frac{E_0}{\eta_1} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

where $\bar{k}_x = \hat{a}_x k_1 \sin\theta_i$, $\bar{k}_z = \hat{a}_z k_1 \cos\theta_i$

$$\bar{E}^r = \hat{a}_y \Gamma_{TE} E_0 e^{-jk_1(x\sin\theta_i - z\cos\theta_i)}$$

$$\bar{H}^r = (\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_r)\Gamma_{TE}\frac{E_0}{\eta_1} e^{-jk_1(x\sin\theta_i - z\cos\theta_i)}$$

Similarly,

$$\bar{E}^t = \hat{a}_y T_{TE} E_0 e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\bar{H}^t = (-\hat{a}_x \cos\theta_t + \hat{a}_z \sin\theta_t) . T_{TE} . \frac{E_0}{\eta_2} e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

Boundary Conditions:

1-) $\bar{E}^i + \bar{E}^r \Big|_{tan, z=0} = \bar{E}^t \Big|_{tan, z=0}$

2-) $\bar{H}^i + \bar{H}^r \Big|_{tan, z=0} = \bar{H}^t \Big|_{tan, z=0}$

There are 2 B.C.'s that give us 4 equations when we equate the real and imaginary parts among themselves. These 4 equations give us the relation among $\theta_r, \theta_t, \Gamma_{TE}$ and T_{TE}

The following relations are obtained:

1-) $\boxed{\theta_r = \theta_i}$ (Snell's law of reflection)

2-) $\boxed{k_1 \sin\theta_i = k_2 \sin\theta_t}$ (Snell's law of refraction or transmission)

k_1 and k_2 can be obtained from the Table in pg. 9

For lossless dielectrics , $k_1 = \omega\sqrt{\mu_1\epsilon_1}$, $k_2 = \omega\sqrt{\mu_2\epsilon_2}$ ($\bar{k}$ = wave number)

Also,

$$\boxed{\Gamma_{TE} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}}$$

$T_{TE} = \frac{2\eta_2\cos\theta_i}{\ldots}$

} (Fresnel's Transmission Coefficients)

$$\eta_2 \cos\theta_i + \eta_1 \cos\theta_t$$

$$T_{TE} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$$

} (Fresnel's Transmission Coefficients.)

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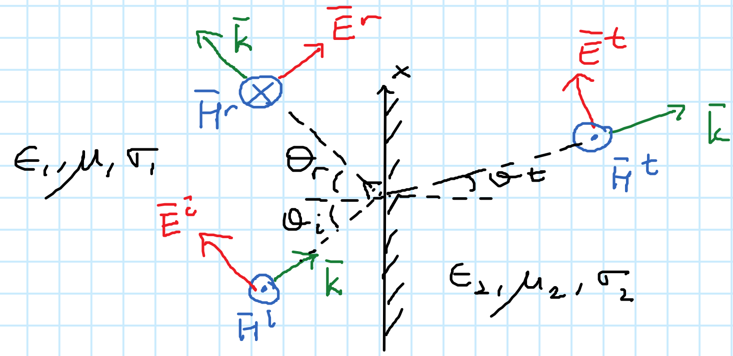
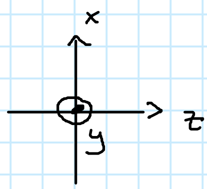
b-) Parallel (TM) Polarization





where

$$\bar{E}^i = (\hat{a}_x \cos\theta_i - \hat{a}_z \sin\theta_i) E_o e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\bar{H}^i = \hat{a}_y \frac{E_o}{\eta} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

and

$$\bar{E}^r = (\hat{a}_x \cos\theta_r + \hat{a}_z \sin\theta_i) \Gamma_{TM} E_o e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\bar{H}^r = -\hat{a}_y \Gamma_{TM} \frac{E_o}{\eta_1} e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

and

$$\bar{E}^t = (\hat{a}_x \cos\theta_t - \hat{a}_z \sin\theta_t) T_{TM} E_o e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

$$\bar{H}^t = \hat{a}_y T_{TM} \frac{E_o}{\eta_2} e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

Also, after imposing the B.C.'s:

1-) $\theta_r = \theta_i$ (Snell's law of reflection)

2-) $k_1 \sin\theta_i = k_2 \sin\theta_t$ (Snell's law of refraction)

and

$$\Gamma_{TM} = \frac{-\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

$$T_{TM} = \frac{2\eta_2 \cos\theta_i}{\eta_1 \cos\theta_i + \eta_2 \cos\theta_t}$$

X

Ex:

A TE polarized plane wave is incident from air to glass interface ($\epsilon_r = 9$) with an angle of incidence $60°$

Find the reflection and transmission coefficients

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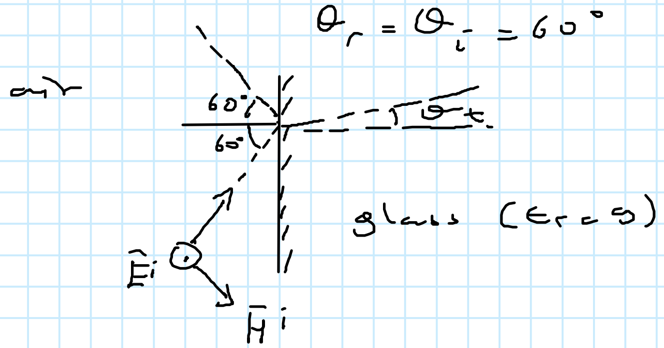
Find the amount of power transfered from medium 1 to medium 2. Given $E_o = 1 \frac{mV}{m}$.

Ans:

From the Snell's law of reflection:

$$\theta_r = \theta_i = 60°$$



From the Snell's law of refraction.

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

where $k_1 = \omega\sqrt{\mu_1\epsilon_1} = \omega\sqrt{\mu_0\epsilon_0}$

and $k_2 = \omega\sqrt{\mu_2\epsilon_2} = \omega\sqrt{\mu_0\epsilon_{r_2}\cdot\epsilon_0} = 3\omega\sqrt{\mu_0\epsilon_0}$ ($\epsilon_{r_2} \Rightarrow 9$)

Substituting k_1 and k_2 into the Snell's law of refraction,

$$\omega\sqrt{\mu_0\epsilon_0}\,\underbrace{\sin\theta_i}_{\frac{\sqrt{3}}{2}} = 3\omega\sqrt{\mu_0\epsilon_0}\,\sin\theta_t$$

$$\Rightarrow \sin\theta_t = \frac{\sqrt{3}}{6} \quad \Rightarrow \theta_t = \sin^{-1}\left(\frac{\sqrt{3}}{6}\right) = 16.78° \approx 17°$$

Also,

$$\Gamma_{TE} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

where

$$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} = \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \sqrt{\frac{\mu_0}{\epsilon_{r_2}\epsilon_0}} = \frac{1}{3}\eta_1$$

$$\Gamma_{TE} = \frac{\frac{1}{3}\eta_1\cos 60° - \eta_1\cos 17°}{\frac{1}{3}\eta_1\cos 60° + \eta_1\cos 17°} = \frac{\frac{1}{3}\cdot(0.5) - 0.9563}{\frac{1}{3}(0.5) + 0.9563}$$

$$\Gamma_{TE} = -0.7$$

$$\Rightarrow 1 + \Gamma = T \quad \Rightarrow 1 - 0.7 = T$$

$$\Rightarrow T_{TE} = 0.3$$

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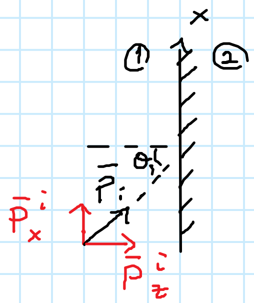
Power Analysis:



$\Gamma_{TE} = -0.7$

$T_{TE} = 0.3$

$$|\bar{P}_{avg}^i| = \frac{1}{2}\,\mathrm{Re}[\bar{E}^i \times \bar{H}^{i*}] = \frac{1}{2}\frac{E_0^2}{\eta_1}\left(\frac{W}{m^2}\right)$$

$$= \frac{1}{2}\frac{(10^{-3})^2}{377} = 1.32\ \frac{nW}{m^2}$$

$$|\bar{P}_{z_{avg}}^i| = |\bar{P}_{avg}^i| \cdot \cos\theta_i = \left(1.32\,\frac{nW}{m^2}\right)\cdot\left(\frac{1}{2}\right) = \boxed{0.66\ \frac{nW}{m^2}}$$

(θ_i = $60°$)

$$|\bar{P}_{avg}^r| = \frac{1}{2}\,\mathrm{Re}[\bar{E}_r \times \bar{H}^{r*}] = \frac{1}{2}\frac{E_0^2}{\eta_1}\cdot|\Gamma|^2 = \frac{1}{2}\frac{(10^{-3})^2}{377}(0.7)^2$$

$$= 0.65\,nW.$$

$$\Rightarrow |\bar{P}_{z_{avg}}^r| = |\bar{P}_{avg}^r|\cos 60° = (0.65\,nW)(0.5) = \boxed{0.325\,nW}$$

$$|\bar{P}_{avg}^t| = \frac{1}{2}\frac{E_0^2}{\eta_1}\cdot\frac{\eta_1}{\eta_2}\cdot|T_{TE}|^2 = (1-|\Gamma|^2)|\bar{P}_{avg}^i| = (1-0.49)\left(1.32\,\frac{nW}{m^2}\right)$$

$$= 0.3564\ \frac{nW}{m^2}$$

$$\Rightarrow |\bar{P}_{z_{avg}}^t| = \left(0.3564\,\frac{nW}{m^2}\right)\cdot\cos(\theta_t)$$

(θ_t = $17°$)

$$= \left(0.3564\,\frac{nW}{m^2}\right)(0.9563) = \boxed{0.3412\ \frac{nW}{m^2}}$$

$$0.325\,\frac{nW}{m^2} + 0.3412\,\frac{nW}{m^2} = 0.66\ \frac{nW}{m^2}$$

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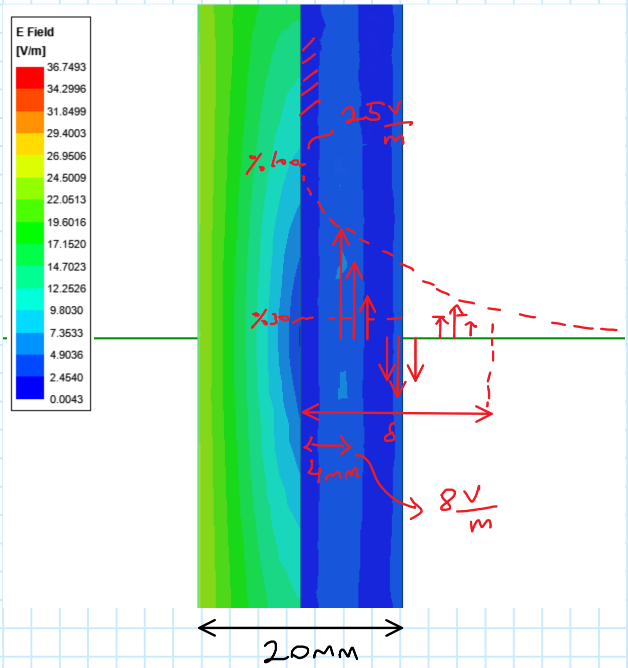
Simulation of air to water interface (normal incidence at $f = 2.4\,GHz$)



Analysis:

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}}$$

$$= \sqrt{\frac{(2\pi \times 2.4\times10^{9})(4\pi\times10^{-7})(5)}{2}}$$

$$= 217.65$$

$$\Rightarrow \delta = \frac{1}{\alpha} = \frac{1}{217.65} = 4.59\ mm$$

⇒ The simulation result is in good agreement with the calculated values.

2 Concepts:

1-) Total Transmission:

a-) TE Polarization

$$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0$$

$$\eta_2 \cos\theta_i - \eta_1 \cos\theta_t = 0$$

or

$$\eta_2 \cos\theta_i = \eta_1 \cos\theta_t$$

or

$$\cos\theta_i = \frac{\eta_1}{\eta_2}\cos\theta_t \ , \quad \eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}} \ \text{ and } \ \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} \ \text{(dielec.)}$$

or

$$\cos\theta_i = \sqrt{\frac{\mu_1}{\mu_2}\left(\frac{\epsilon_2}{\epsilon_1}\right)} \cdot \cos\theta_t \quad \text{—(1)}$$

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We have also, Snell's law of refraction

$$k_1 \sin\theta_i = k_2 \sin\theta_t$$

or

$$\sin\theta_i = \frac{k_2}{k_1}\sin\theta_t$$

or

$$\sin^2\theta_i = \left(\frac{k_2}{k_1}\right)^2 \sin^2\theta_t \quad , \; k_1 = \omega\sqrt{\mu_1\epsilon_1} \; , \; k_2 = \omega\sqrt{\mu_2\epsilon_2}$$

or

$$\sin^2\theta_i = \left(\frac{\omega\sqrt{\mu_2\epsilon_2}}{\omega\sqrt{\mu_1\epsilon_1}}\right)^2 \sin^2\theta_t$$

or

$$\sin^2\theta_i = \frac{\mu_2}{\mu_1}\frac{\epsilon_2}{\epsilon_1}\cdot\sin^2\theta_t$$

or

$$\sin^2\theta_t = \frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_i \quad \text{——(2)}$$

Then, we have

$$\cos\theta_i = \sqrt{\frac{\mu_1}{\mu_2}\left(\frac{\epsilon_2}{\epsilon_1}\right)}\cdot\cos\theta_t \quad \text{——(1)}$$

Take the square of both sides of (1).

$$\cos^2\theta_i = \frac{\mu_1}{\mu_2}\frac{\epsilon_2}{\epsilon_1}\cos^2\theta_t$$

or

$$1-\sin^2\theta_i = \frac{\mu_1}{\mu_2}\frac{\epsilon_2}{\epsilon_1}\left(1-\underline{\sin^2\theta_t}\right)$$

↳ replace this term by eqn. (2)

Then,

$$1-\sin^2\theta_i = \frac{\mu_1}{\mu_2}\frac{\epsilon_2}{\epsilon_1}\left(1-\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\sin^2\theta_i\right)$$

Solving for $\sin\theta_i$ gives

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1}-\frac{\mu_2}{\mu_1}}{\frac{\mu_1}{\mu_2}-\frac{\mu_2}{\mu_1}}}$$

For θ_i to exist, the term inside the square root must be less than 1, i.e

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$$\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1} \le \frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1}$$

or

$$\boxed{\frac{\epsilon_2}{\epsilon_1} \le \frac{\mu_1}{\mu_2}} \quad \text{must hold}$$

If $\mu_1 = \mu_2 = \mu_0 \Rightarrow$

$$\boxed{\epsilon_2 \le \epsilon_1}$$

However, for $\mu_1 = \mu_2 = \mu_0$:

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1}}} = \infty \qquad \left(\frac{\mu_1}{\mu_2} - \frac{\mu_2}{\mu_1} \to 0\right)$$

$\sin\theta_i = \infty$ means that there is no solution $\Rightarrow$ No such angle exists

b-) TM Pol

$$\Gamma_{TM} = \frac{-\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_i + \sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_t}{\sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_i + \sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_t} = 0$$

If we go through a similar analysis as before

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1}}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}}}$$

$$\Rightarrow \frac{\epsilon_2}{\epsilon_1} - \frac{\mu_2}{\mu_1} \le \frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2} \Rightarrow \boxed{\frac{\mu_2}{\mu_1} \ge \frac{\epsilon_1}{\epsilon_2}} \quad \text{must hold.}$$

If $\mu_1 = \mu_2 = \mu_0 \Rightarrow$

$$\sin\theta_i = \sqrt{\frac{\frac{\epsilon_2}{\epsilon_1} - 1}{\frac{\epsilon_2}{\epsilon_1} - \frac{\epsilon_1}{\epsilon_2}}} = \sqrt{\frac{\frac{\epsilon_2 - \epsilon_1}{\epsilon_1}}{\frac{\epsilon_2^2 - \epsilon_1^2}{\epsilon_1\epsilon_2}}}$$

or

$$\sin\theta_i = \left[\frac{\epsilon_2 - \epsilon_1}{\epsilon_1} \cdot \frac{\epsilon_1\epsilon_2}{(\epsilon_2 - \epsilon_1)(\epsilon_2 + \epsilon_1)}\right]^{\frac{1}{2}} = \sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}$$

$$\Rightarrow \boxed{\theta_i = \sin^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1 + \epsilon_2}}\right)}$$

(Angle of total transmission or Brewster angle)

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Also it can be written as

$$\theta_i = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$$

- Thus, the 2nd medium must be denser. ↳ in terms of polarization

$$1 = \frac{\mu_0}{\mu_0} \geq \frac{\epsilon_1}{\epsilon_2}$$

$$\Rightarrow \epsilon_2 \geq \epsilon_1$$

Ex:

A TM pol. plane wave is incident from air to polystere ($\epsilon_r = 2.56$) interface with oblique incidence. Find the angle of incidence such that all the wave energy is transmitted.

Ans.

$\epsilon_1 = \epsilon_0$, $\epsilon_2 = 2.56\,\epsilon_0$.

$$\Rightarrow \theta_i = \sin^{-1}\left(\sqrt{\frac{2.56\,\epsilon_0}{\epsilon_0(1+2.56)}}\right) = \sin^{-1}\left(\sqrt{\frac{2.56}{3.56}}\right) = 58^\circ //.$$

Alternatively, $\theta_i = \tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \tan^{-1}\left(\sqrt{\frac{2.56\,\epsilon_0}{\epsilon_0}}\right) = 58^\circ //.$

2-) Total Reflection

a-) TE Pol.

$$|\Gamma_{TE}| = \left| \frac{\sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_i - \sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t}{\sqrt{\frac{\mu_2}{\epsilon_2}}\cos\theta_i + \sqrt{\frac{\mu_1}{\epsilon_1}}\cos\theta_t} \right| = 1 \quad \text{—— (1)}$$

Also,

$$\cos\theta_t = \sqrt{1 - \sin^2\theta_t} = \sqrt{1 - \frac{\mu_1}{\mu_2}\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i}$$

($\sin^2\theta_t$ →) Snell's law of refraction

For θ_t to not exist

$$\frac{\mu_1}{\mu_2}\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i \geq 1$$

This condition makes $\cos\theta_t$ complex and no solution exists for θ_t

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$$\frac{\mu_1}{\mu_2}\frac{\epsilon_1}{\epsilon_2}\sin^2\theta_i \geq 1$$

Solving for θ_i gives

$$\boxed{\theta_i \geq \sin^{-1}\left(\sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}\right)}$$ (condition for all reflection.)

for when $\theta_i = \sin^{-1}\left(\sqrt{\frac{\mu_2\epsilon_2}{\mu_1\epsilon_1}}\right)$ is called "critical angle".

The condition

$$\boxed{\mu_2\epsilon_2 \leq \mu_1\epsilon_1}$$ to θ_i to exist

For $\mu_1=\mu_2=\mu_0$

$$\boxed{\theta_i = \sin^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)}$$ critical angle

and the condition becomes

$$\boxed{\epsilon_2 \leq \epsilon_1}$$

– The 1st medium must be denser

b-) TM pol.

The critical angle formulations and equations and the conditions are the same as in TE pol.

– Thus, critical angle is polarization independent

Ex:

A TE pol. wave is incident at glass to air interface ($\epsilon_r = 9$ for glass). Find the angle for all waves to be reflected.

Ans.

$\epsilon_1 = 9\epsilon_2$, $\epsilon_2 = \epsilon_0$

$$\Rightarrow \theta_i = \theta_c = \sin^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right) = \sin^{-1}\left(\sqrt{\frac{1}{9}}\right) = \sin^{-1}\left(\frac{1}{3}\right) \approx 20^\circ.$$

If $\theta_i \geq \theta_c \Rightarrow$ all energy reflects.

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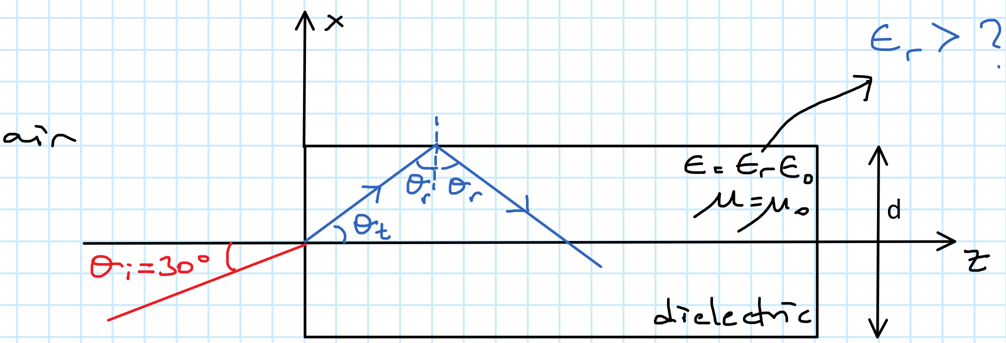
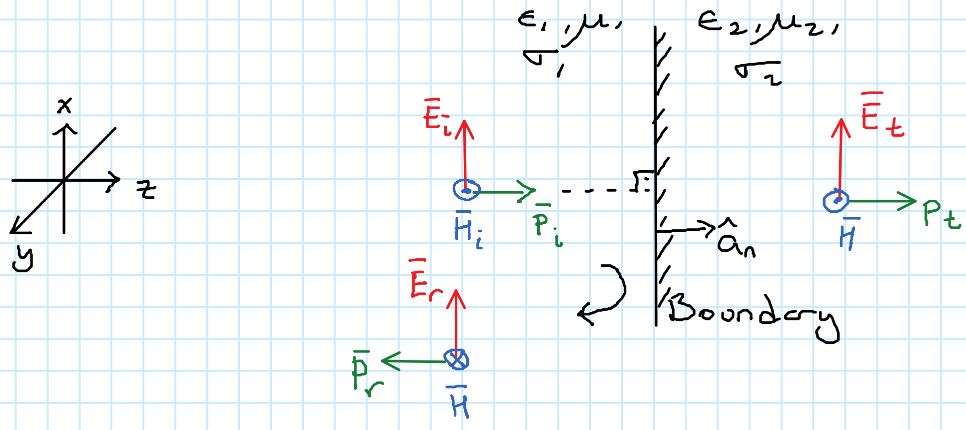
HW #7:

For a given dielectric cylindrical structure with diameter d, the plane wave enters the structure from air from its one end. For $\theta_i = 30°$, find the required range of dielectr. constant ϵ_r for the dielectric material so that the energy is retained inside the structure.



— Normal and Oblique Incidences in Lossy Media —

— Normal Incidence:



Writing the expressions for the fields

$$\bar{E}_i = \hat{a}_x E_0 e^{-\alpha_1 z} e^{-j\beta_1 z} \quad \text{(Phasor)}$$

$$\bar{H}_i = \hat{a}_y \frac{E_0}{\eta_1} e^{-\alpha_1 z} e^{-j\beta_1 z}$$

and

$$\bar{E}_r = \hat{a}_x \Gamma . E_0 \, e^{+\alpha_1 z} e^{j\beta_1 z}$$

$$\bar{H}_r = -\hat{a}_y \Gamma . \frac{E_0}{\eta} e^{\alpha_1 z} e^{j\beta_1 z}$$

Note. For lossy media,

$\gamma = \alpha + j\beta$ ← Phase constant

For lossless media,

$\alpha = 0$, k = wavenumber = β

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and

$$\bar{E}_t = \hat{a}_x T \cdot E_o e^{-\alpha_2 z} \cdot e^{-j\beta_2 z} \Rightarrow \bar{E}_t(z,t) = \hat{a}_x T E_o e^{-\alpha_2 z} \cos(\omega t - \beta_2 z)$$

$$\bar{H}_t = \hat{a}_y T \frac{E_o}{\eta_2} e^{-\alpha_2 z} \cdot e^{-j\beta_2 z}$$

and

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$, where η_1, η_2 are complex from the table in pg. 17.

$$T = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Power Analysis

$$S^i_{avg} = \hat{a}_z \frac{|E_o|^2}{2} e^{-2\alpha_1 z} \mathrm{Re}\left(\frac{1}{\eta_1^*}\right)$$

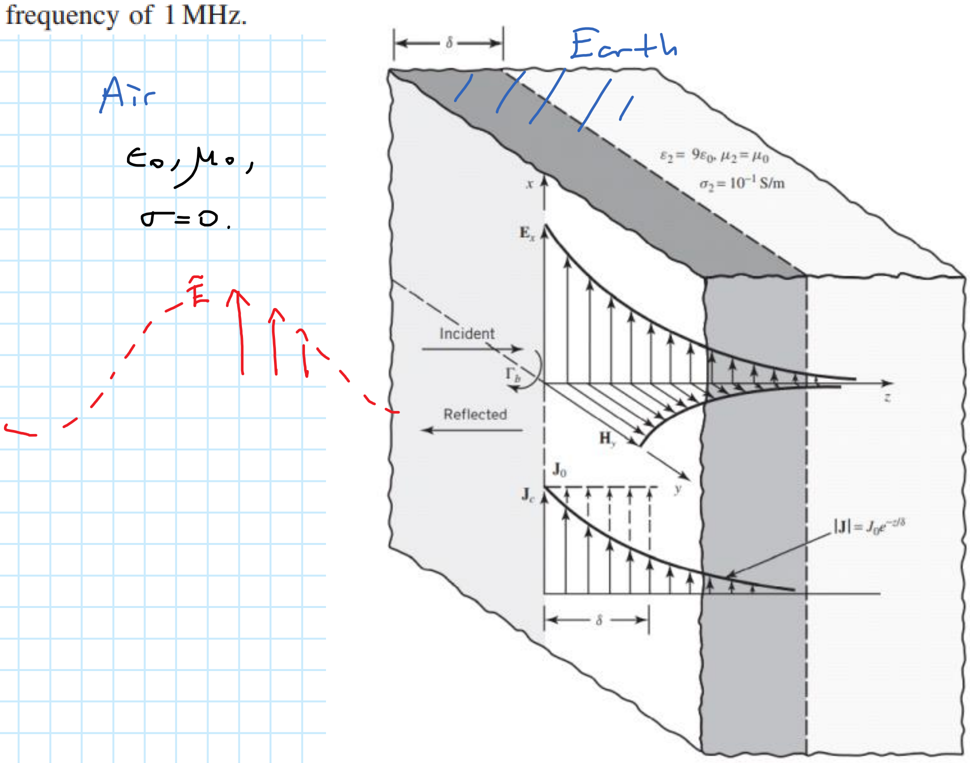
$$S^r_{avg} = -\hat{a}_z |\Gamma|^2 \frac{|E_o|^2}{2} e^{+2\alpha_1 z} \mathrm{Re}\left(\frac{1}{\eta_1^*}\right)$$

$$S^t_{avg} = \hat{a}_z |T|^2 \frac{|E_o|^2}{2} e^{-2\alpha_2 z} \mathrm{Re}\left(\frac{1}{\eta_2^*}\right)$$

where $\alpha_1, \alpha_2, \beta_1, \beta_2, \eta_1$ and η_2 can be computed from the table in pg 17 with constants $\epsilon_1, \epsilon_2, \mu_1, \mu_2$ and σ_1 and σ_2 are given.

Ex

A uniform plane wave, whose incident electric field has an x component with an amplitude at the interface of 10^{-3} V/m, is traveling in a free-space medium and is normally incident upon a lossy flat earth as shown in Figure ~~X~~. Assuming that the constitutive parameters of the earth are $\varepsilon_2 = 9\varepsilon_0$, $\mu_2 = \mu_0$ and $\sigma_2 = 10^{-1}$ S/m, determine the variation of the conduction current density in the earth at a frequency of 1 MHz.



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Ans

- Find $\bar{E}_t$
- $\bar{J}_c = \sigma_2 \cdot \bar{E}_t$ (Ohm's law)

$\Rightarrow$ We need to evaluate $T = \dfrac{2\eta_2}{\eta_2 + \eta_1}$ where $\eta_1 = 377\,\Omega$ and

to find η_2:

We check: $\left(\dfrac{\sigma_2}{\omega \epsilon_2}\right)^2 = \left(\dfrac{10^{-1}}{2\pi(10^6) 9.(8.854 \times 10^{-12})}\right)^2 = 40000 \gg 1. \Rightarrow$ Good conductor

$$\Rightarrow \eta_2 = \sqrt{\frac{\omega\mu}{2\sigma}}\,(1+j) = \sqrt{\frac{(2\pi \times 10^6)(4\pi \times 10^{-7})}{2 \times 10^{-1}}}\,(1+j) = 2\pi(1+j)$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{2\pi(1+j) - 377}{2\pi(1+j) + 377} = 0.967\angle 178.1^\circ$$

and

$$T = 1 + \Gamma = 1 + 0.967\angle 178.1^\circ$$

Thus,

$$\bar{E}_t = \hat{a}_x T \cdot E_0\, e^{-\alpha_2 z} \cdot e^{-j\beta_2 z} \qquad \text{where } E_0 = 1\,\frac{mV}{m}.$$

$$\bar{H}_t = \hat{a}_y T \frac{E_0}{\eta_2}\, e^{-\alpha_2 z} \cdot e^{-j\beta_2 z}$$

$$\Rightarrow |\bar{E}_t| = |T| \cdot |E_0| = 10^{-3}\,|0.0335 + j0.0321| = 4.64 \times 10^{-5}\,\frac{V}{m}$$

$$\bar{J}_c\Big|_{z=0} = \sigma|\bar{E}_t| = 10^{-1}\,\frac{s}{m}\left(4.64 \times 10^{-5}\,\frac{V}{m}\right) = 4.64 \times 10^{-6}\,\frac{V \cdot s}{m^2}$$

$$\Rightarrow J_0 = 4.64\,\mu\frac{A}{m^2}$$

$$|\bar{J}_c(z)| = J_0 \cdot e^{-\alpha_2 z} = J_0\, e^{-\frac{z}{\delta}}$$

where

$$\delta = \sqrt{\frac{2}{\omega\mu_2\sigma_2}} = \sqrt{\frac{2}{2\pi \times 10^6 (4\pi \times 10^{-7})\, 10^{-1}}} = 1.59\,m.$$

$$\Rightarrow |\bar{J}_c(z)|\Big|_{z=1.59\,m} = J_0 \cdot e^{-1} = 0.3679 \cdot J_0 = 0.3679\left(4.64\,\mu\frac{A}{m^2}\right) = 1.707\,\mu\frac{A}{m^2}$$

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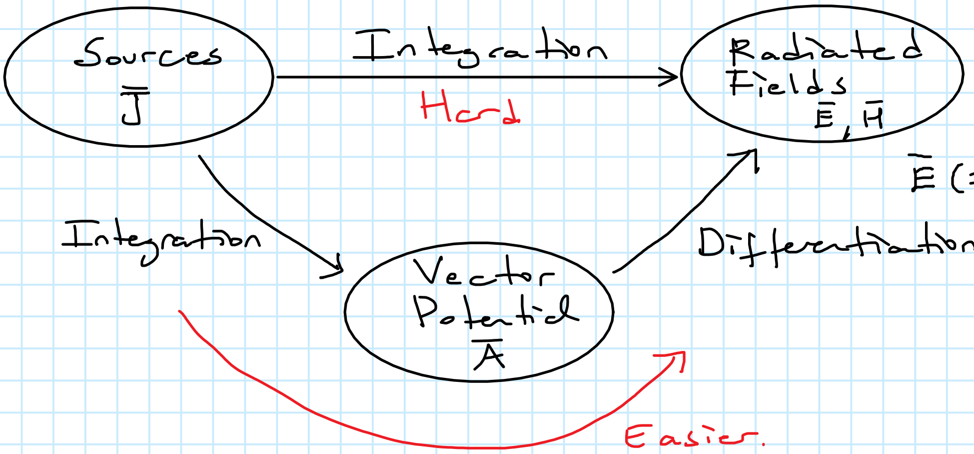
- Oblique Incidence:

The formulations of the oblique incidence in lossy media are the same as that of the lossless media with α, β, η and other parameters are functions of σ as in table in p5-9.

- Radiation -

The problem of radiation can be depicted as in the figure below



$\bar{E}(z) = \hat{a}_x E_o e^{-jkz}$

Differentiation Path:

$$\bar{E} = -j\omega\bar{A} - j\frac{1}{\omega\mu\varepsilon}\bar{\nabla}(\bar{\nabla}\cdot\bar{A})$$

Integration Path:

$$\bar{A} = \frac{\mu}{4\pi}\int_V \bar{J}\cdot\frac{e^{-jkR}}{R}\,dv' \quad , \quad \bar{J} = \frac{A}{m^2} \quad \text{(volumetric source)}$$

or for surface currents (surface source)

$$\bar{A} = \frac{\mu}{4\pi}\int_S \bar{J}_s\cdot\frac{e^{-jkR}}{R}\,ds' \quad , \quad \bar{J}_s = \frac{A}{m} \quad \text{(surface source)}$$

or for linear current sources:

$$\bar{A} = \frac{\mu}{4\pi}\int_C \bar{I}\cdot\frac{e^{-jkR}}{R}\,dl' \quad , \quad I = A \quad \text{(current)}$$

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} dl' \quad , \quad I = A \text{ (current)}$$

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For field approximations $(R \gg L)$

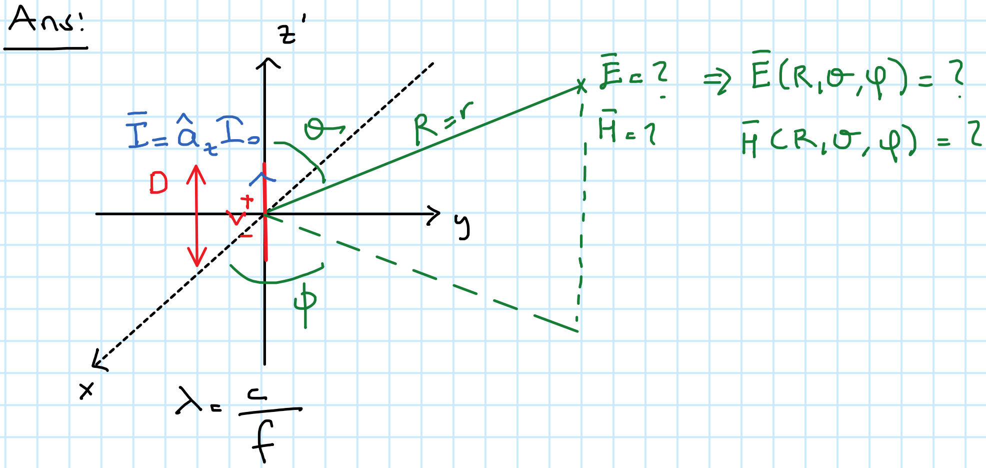
$R = r$ (radial distance from the origin)

$\bar{E} = -j\omega \bar{A}$ for E_θ component only $E_R = 0$

Ex:

Given a very small conductor $(D \ll \lambda)$ with a current $\bar{I} = \hat{a}_z I_o$ (phasor), find the radiated fields in air?

Ans:



To find A:

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} dl'$$

or

$$\bar{A} = \frac{\mu_o}{4\pi} \int_C (\hat{a}_z I_o) \frac{e^{-jkr}}{r} dz'$$

$$\Rightarrow \bar{A} = \frac{4\pi \times 10^{-7}}{4\pi} \int_{-D/2}^{D/2} \hat{a}_z I_o \frac{e^{-jkr}}{r} dz'$$

$A_\theta = A_z \sin\theta$

$$\Rightarrow \bar{A} = \hat{a}_z 10^{-7} \cdot I_o \cdot \frac{e^{-jkr}}{r} D.$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}_\theta \Rightarrow \bar{E}_\theta = -j\omega 10^{-7} \cdot I_o \frac{e^{-jkr}}{r} \cdot D \cdot \sin\theta \quad \left(\frac{V}{m}\right)$$

or

$$E_\theta = 10^{-7} \omega I_o \cdot D \cdot \sin\theta \frac{e^{-jkr}}{r} e^{-j\frac{\pi}{2}}$$

$$\bar{E}_\theta(r,t) = \mathrm{Re}\left[E_\theta \cdot e^{j\omega t}\right] = \underbrace{10^{-7} \omega I_o \cdot D \cdot \frac{1}{r}}_{E_o} \sin\theta \cos(\omega t - \underbrace{kr}_{\text{Phase factor}} - \frac{\pi}{2})$$

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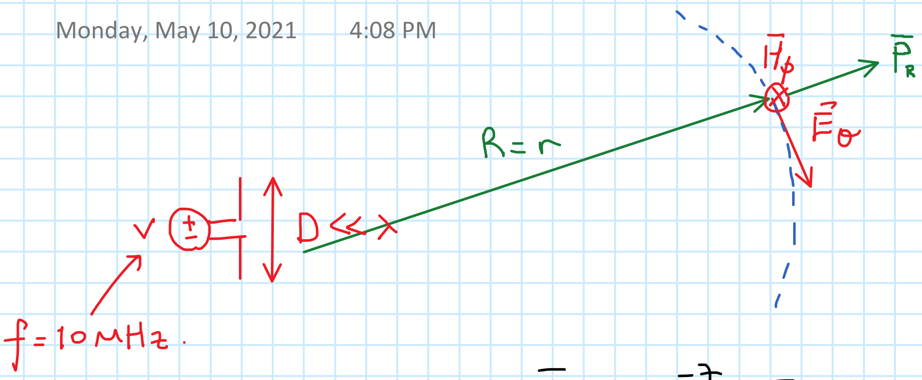
Monday, May 10, 2021 4:08 PM



$\lambda\big|_{f=10\,MHz} = \frac{c}{f} = \frac{3\times10^8}{10\times10^6} = 30$

$D \ll 30\,m$. $D = 1\,m$ ✓

$\bar{E}_\theta = 10^{-7}\,\omega\, I_0 \cdot D \cdot \frac{e^{-jkr}}{r}\, e^{-j\frac{\pi}{2}}$ (Phasor)

$E_\theta = \underbrace{10^{-7}\,\omega\, I_0 \cdot D \cdot \frac{1}{r}}_{E_0} \cos(\omega t - kr - \underbrace{\tfrac{\pi}{2}}_{\text{phase factor}})$

$\Rightarrow \bar{H}(r) = \frac{1}{\eta}\, \hat{a}_n \times \bar{E}(r)$

$= \frac{1}{377} \cdot \hat{a}_R \times \bar{E}_\theta$

$= \hat{a}_\phi\, \frac{10^{-7}\,\omega\, I_0}{377}\, D\, \frac{1}{r} \cos(\omega t - kr - \frac{\pi}{2})$

$\bar{P} = \bar{E} \times \bar{H}$ (W/m^2) $\Rightarrow$ in $\hat{a}_R$ direction

Ex:

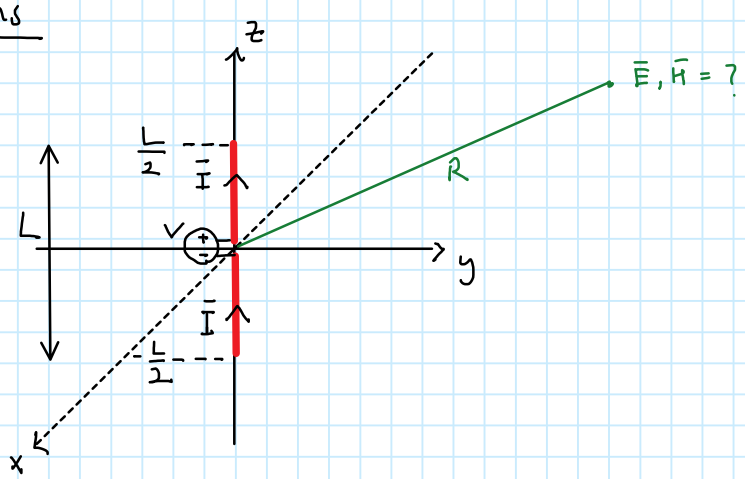
Finite length conductor is used

The current density is given as

$$\bar{I}(z') = \begin{cases} \hat{a}_z\, I_0 \sin\left[k\left(\frac{L}{2} - z'\right)\right], & 0 \leq z' \leq L/2 \\ \hat{a}_z\, I_0 \sin\left[k\left(\frac{L}{2} + z'\right)\right], & -\frac{L}{2} \leq z' \leq 0 \end{cases}$$

Find the radiated fields at far away from the conductor. frequency = 1 GHz.

Ans



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Analysis of I(z')

$$\bar{I}(z') = \begin{cases} \hat{a}_z I_o \sin\left[k\left(\frac{L}{2} - z'\right)\right], & 0 \leq z' \leq L/2 \\ \hat{a}_z I_o \sin\left[k\left(\frac{L}{2} + z'\right)\right], & -\frac{L}{2} \leq z' \leq 0 \end{cases}$$

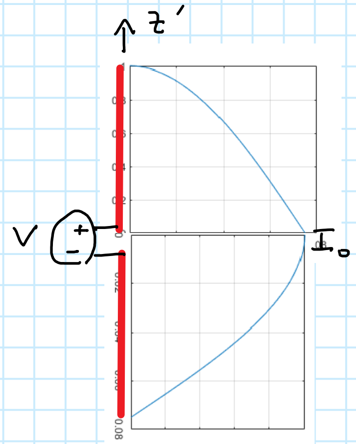
$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^9} = 0.3 = 30\,cm$$

$$k = \text{wave number} = \frac{2\pi}{\lambda} = \frac{2\pi}{0.3} = \frac{2\pi}{\frac{3}{10}} = \frac{20\pi}{3}$$

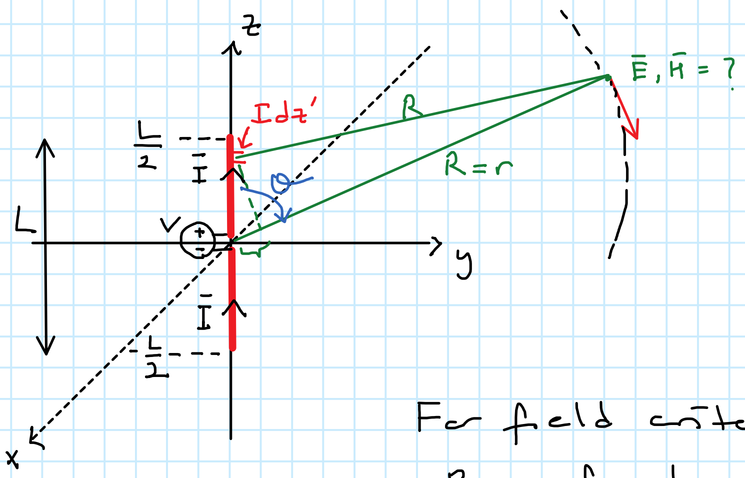
L = Conductor length $= \frac{\lambda}{2} = 15\,cm$ (Half-wave dipole)

$$\Rightarrow \bar{I}(z') = \begin{cases} \hat{a}_z I_o \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{2} - z'\right)\right], & 0 \leq z' \leq L/2 \\ \hat{a}_z I_o \sin\left[\frac{2\pi}{\lambda}\left(\frac{\lambda}{2} + z'\right)\right], & -\frac{L}{2} \leq z' \leq 0 \end{cases}$$

(Phasor)



Finding $\bar{A}$ from $\bar{I}(z')$



$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} dl'$$

$$\Rightarrow \bar{A} = \frac{\mu}{4\pi} \int_0^{L/2} \bar{I}(z') \frac{e^{-jkR}}{R} dz' + \frac{\mu}{4\pi} \int_{-\frac{L}{2}}^{0} \bar{I}(z') \frac{e^{-jkR}}{R} dz'$$

Far field criteria: $R \gg L$.

$R = r$ for denominator term

$R = r - z'\cos\theta$ for the phase term

The integral gives $\bar{A} = \hat{a}_z \eta \frac{I_o e^{-jkr}}{w 2\pi r} \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right]$

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and $\bar{E} = -j\omega A$

$$\Rightarrow E_\theta = j\eta \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] \left(\frac{V}{m}\right)$$

$$H_\phi = j \frac{I_o e^{-jkr}}{2\pi r}\left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] \left(\frac{A}{m}\right)$$

$$\bar{P}_{avg} = \hat{a}_R \left(\eta \frac{I_o}{2\pi r}\right)^2 \Big/ 2\eta \left(\frac{W}{m^2}\right) \qquad \bar{P}_{avg} = \frac{|E_o|^2}{2\eta}$$

↳ This is the average power

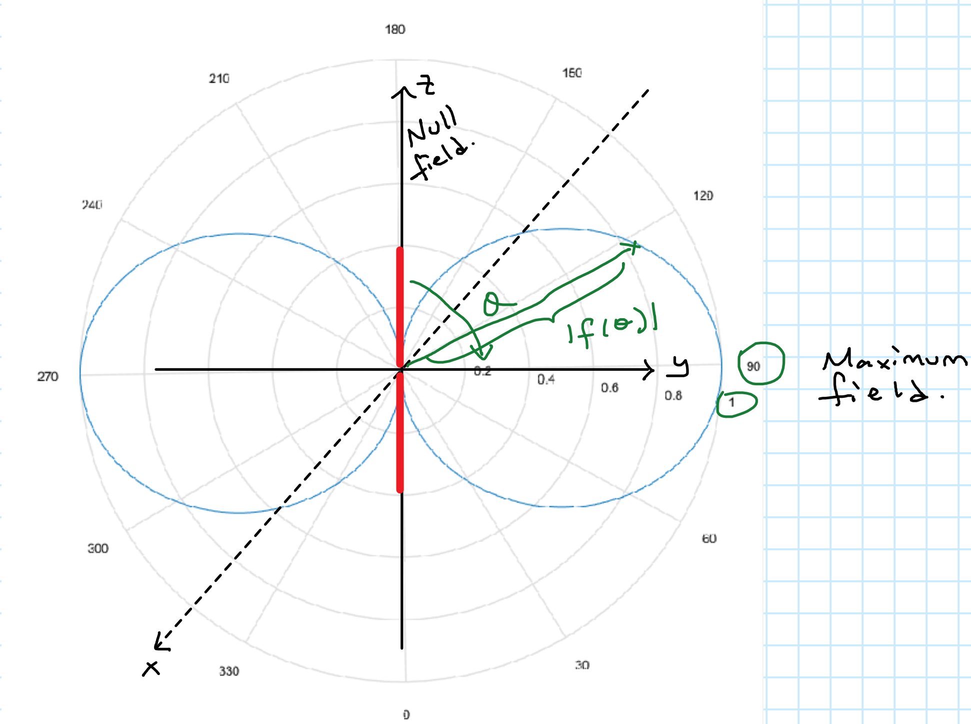
assuming that

$$f(\theta) = \left[\frac{\cos\left(\frac{kL}{2}\cos\theta\right) - \cos\frac{kL}{2}}{\sin\theta}\right] = 1 \text{ (maximum)}$$

where $f(\theta)$ is the part of the field containing angular dependence.

$\Rightarrow f(\theta)$ = Pattern function (Radiation pattern function)

Pattern function: $f(\theta)$



$f(\theta)$ for $L = \frac{\lambda}{2}$ (Half-wave dipole)

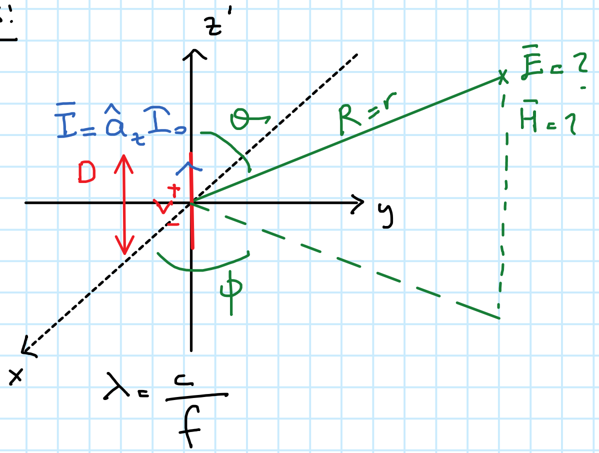
P40

Wednesday, July 28, 2021 11:47 AM

Ex:

Given a very small conductor ($D \ll \lambda$) with a current $\bar{I} = \hat{a}_z I_o z^2$ (phasor), find the radiated fields in air?

Ans:



$\Rightarrow \bar{E}(R, \theta, \varphi) = ?$

$\bar{H}(R, \theta, \varphi) = ?$

To find A:

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} dl'$$

or

$$\bar{A} = \frac{\mu_o}{4\pi} \int_C (\hat{a}_z I_o) \frac{e^{-jkr}}{r} dz'$$

$$\int_{-D/2}^{D/2} z^2 dz = \frac{1}{3} z^3 \Big|_{-D/2}^{D/2} = \frac{1}{12} D^3$$

$$\Rightarrow \bar{A} = \frac{4\pi \times 10^{-7}}{4\pi} \int_{-D/2}^{D/2} \hat{a}_z I_o z^2 \frac{e^{-jkr}}{r} dz'$$

$$\Rightarrow \bar{A} = \hat{a}_z 10^{-7} \cdot I_o \cdot \frac{e^{-jkr}}{r} \frac{D^3}{12}$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}_\theta \Rightarrow \bar{E}_\theta = -j\omega 10^{-7} \cdot I_o \frac{e^{-jkr}}{r} \cdot \frac{D^3}{12} \sin\theta \left(\frac{V}{m}\right)$$

or

$$E_\theta = 10^{-7} \omega I_o \cdot D \cdot \sin\theta \frac{e^{-jkr}}{r} e^{-j\frac{\pi}{2}}$$

$$E_\theta(r,t) = Re\left[E_\theta \cdot e^{j\omega t}\right] = \underbrace{10^{-7} \omega I_o \cdot \frac{D^3}{12} \cdot \frac{1}{r} \cdot \sin\theta}_{E_o} \cos\left(\omega t - kr - \frac{\pi}{2}\right)$$

$f(\theta)$ = Pattern function ($\sin\theta$)

$-kr$: phase factor

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Wednesday, July 28, 2021 12:01 PM

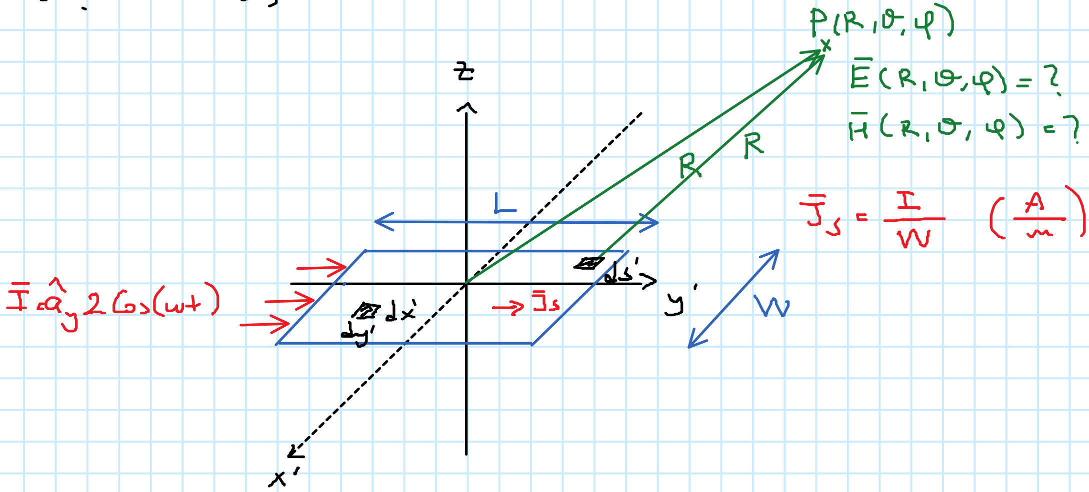
Midterm Exam Information:

- Midterm exam will be held on 06/08/2021 Friday at 13:30 usual lecture time.
- The exam will be open notes.
- Mobile phones and computers are not allowed to be used during the exam. However, you may use a standart calculator.
- The exam will be online with open-camera.
- During the exam, your hands and paper must be visible on the screen.

Ex: X

Given a rectangular sheet (small ($D \ll \lambda$)) with dimensions W and L, a current of 2A at f=1GHz is fed from its one end. Find the radiated $\bar{E}$ and $\bar{H}$ fields? (W=2, L=2)

Ans:



$$\Rightarrow \bar{J}_s = \hat{a}_y \frac{I}{W} = \hat{a}_y 1 \left(\frac{A}{m}\right)$$

First, find $\bar{A}$ from $\bar{J}_s$:

$$\bar{A} = \frac{\mu}{4\pi} \int_S \bar{J}_s \cdot \frac{e^{-jkR}}{R} ds' \quad (ds' \to dx'dy')$$

If $R \gg D = WL$

$\Rightarrow R = r$ (constant)

$$\Rightarrow \bar{A} = \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{W}{2}}^{\frac{W}{2}} \hat{a}_y \cdot 1 \cdot \frac{e^{-jkr}}{r} dx' dy'$$

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$$\bar{A} = \frac{\mu_0}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} \hat{a}_y \cdot 1 \cdot \frac{e^{-jkr}}{r} \, dx' \, dy'$$

$$\Rightarrow \bar{A} = \hat{a}_y \frac{\mu_0}{4\pi} \frac{e^{-jkr}}{r} \underbrace{\int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{w}{2}}^{\frac{w}{2}} dx' \, dy'}_{w \cdot L = 4 m^2}$$

$$\bar{A} = \hat{a}_y \underbrace{\frac{\mu_0}{\pi} \frac{e^{-jkr}}{r}}_{A_y}$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}$$

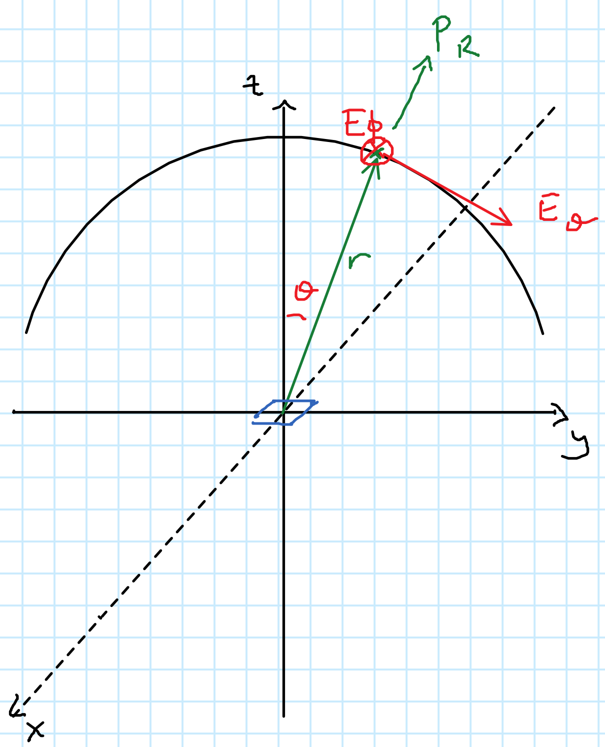
↑ θ or ϕ coord.

Thus, $\boxed{A_\theta = A_y \cos\theta \sin\phi} = \frac{\mu_0}{\pi} \frac{e^{-jkr}}{r} \cos\theta \sin\phi$, $\boxed{A_\phi = A_y \cos\phi}$ $= \frac{\mu_0}{\pi} \frac{e^{-jkr}}{r} \cos\phi$.

$$\Rightarrow E_\theta = -j\omega A_\theta \quad , \quad E_\phi = -j\omega A_\phi$$

$$E_\theta = \frac{\mu_0}{\pi} \underbrace{\cos\theta \sin\phi}_{f(\theta,\varphi)} \frac{e^{-jkr}}{r} \left(\frac{V}{m}\right) \quad \text{(Phasor)}$$

$$E_\phi = \underbrace{\frac{\mu_0}{\pi} \underbrace{\cos\phi}_{f(\varphi)}}_{E_0} \frac{e^{-jkr}}{r} \left(\frac{V}{m}\right) \quad \text{(Phasor)}$$

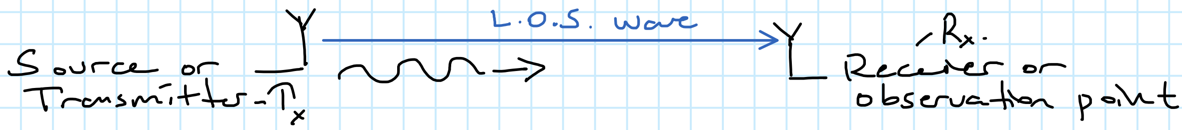


- Electromagnetic Wave Propagation -

In general, there are 3 types of wave propagation.

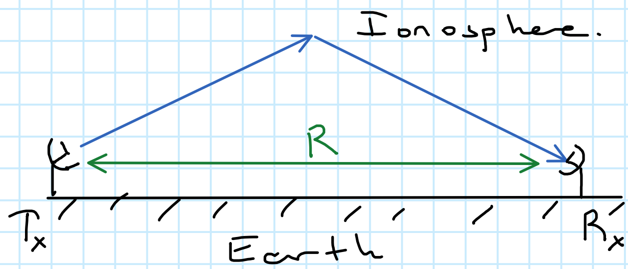
1-) Direct Wave: (Line of Sight Propagation)

- In this propagation, there are no obstacles in between the transmitter and the receiver antennas



2-) Sky Wave (Ionospheric Propagation)

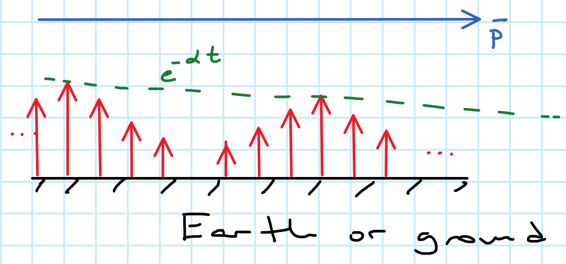
- The source is directed towards the sky, and the waves are reflected from the ionosphere



- Long range communication is possible (R is very large)
- Waves are reflected from the ionosphere

3-) Ground Wave (Surface Waves) Propagation.

- These waves can be parallel or perpendicular to ground
- Ground waves move parallel to earth surface with attenuation



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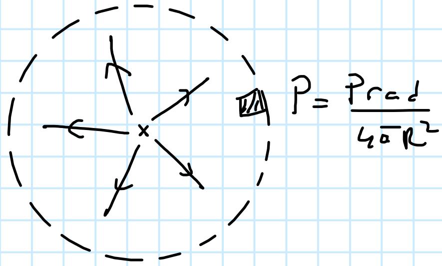
Monday, May 24, 2021 3:53 PM

1-) Direct Wave Propagation:

The power density of an isotropic (equally in all directions) source is:

$$P = \frac{P_{rad}}{4\pi R^2} \left(\frac{W}{m^2}\right)$$

Isotropic radiated power density (W/m^2)



P_{rad} = Total radiated power (W)

In reality the sources (antennas) are not isotropic. They radiate more power in certain directions than the others.

Power density of such a source is:

$$P(\theta, \varphi) = \frac{P_{rad}}{4\pi R^2} \cdot G(\theta, \varphi) \left(\frac{W}{m^2}\right)$$

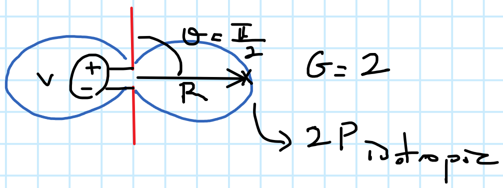
P_{iso} = Isotropic rad power density

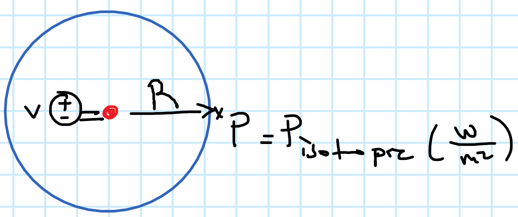
Gain of the source in θ, φ direction

- Unless stated otherwise, θ and φ are the angles which make $G(\theta, \varphi)$ maximum

$$\text{Antenna Gain} = \frac{P(\theta, \varphi)}{P_{isotropic}}$$

Ex. If Gain = G = 2 means that at θ and φ for which G is maximum, the power density of the source is twice that of the isotropic antenna





P45

Monday, May 24, 2021 4:15 PM

Ex:

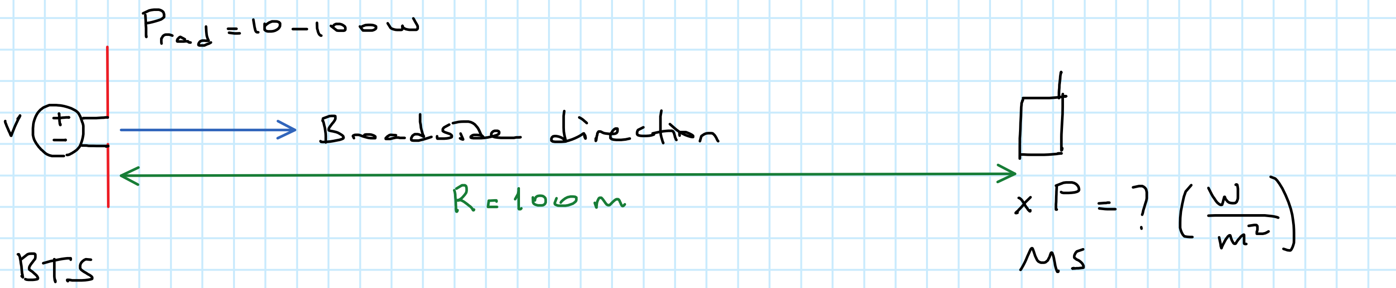
A gsm base station antenna radiates E.M. waves with 10 - 100 W of total output power. Evaluate the power density of the waves at R = 100 m away from this antenna in the broadside direction where the antenna has 12-18 dB for BTS and 2-5 dB for MS.

BTS: Base transceiver station MS: Mobile station.

Use $G_{BTS} = 12$ dB

Note: All the given values are typical.

Ans



$$P = P_{iso} \cdot G(\theta = \tfrac{\pi}{2}) = \frac{P_{rad}}{4\pi R^2} \cdot G(\theta = \tfrac{\pi}{2})$$

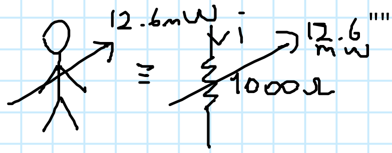
where

$$G_{dB}(\theta = \tfrac{\pi}{2}) = 12\ dB = 10 \log_{10} \frac{G}{G_{ref}}, \quad G_{ref} = 1$$

$$12 = 10 \log_{10} G \Rightarrow G = 10^{1.2}$$

$$\Rightarrow P = \frac{10}{4\pi(100)^2} \cdot 10^{1.2} = 1.26\ \frac{mW}{m^2}$$

$$P = \frac{100}{4\pi(100)^2} \cdot 10^{1.2} = 12.6\ \frac{mW}{m^2}$$

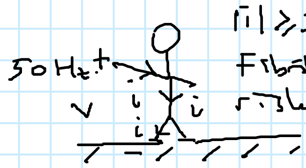


$$P_{avg} = \frac{1}{2} |i|^2 R$$

$$12.6\,mW = \frac{1}{2} \cdot |i|^2 \cdot 1000$$

$$\frac{2 \cdot 6\,mW}{1000} = |i|^2$$

$$\Rightarrow |i| = 1.54\,mA.$$



$|i| \geq 30\,mA$ Fibrilation risk!

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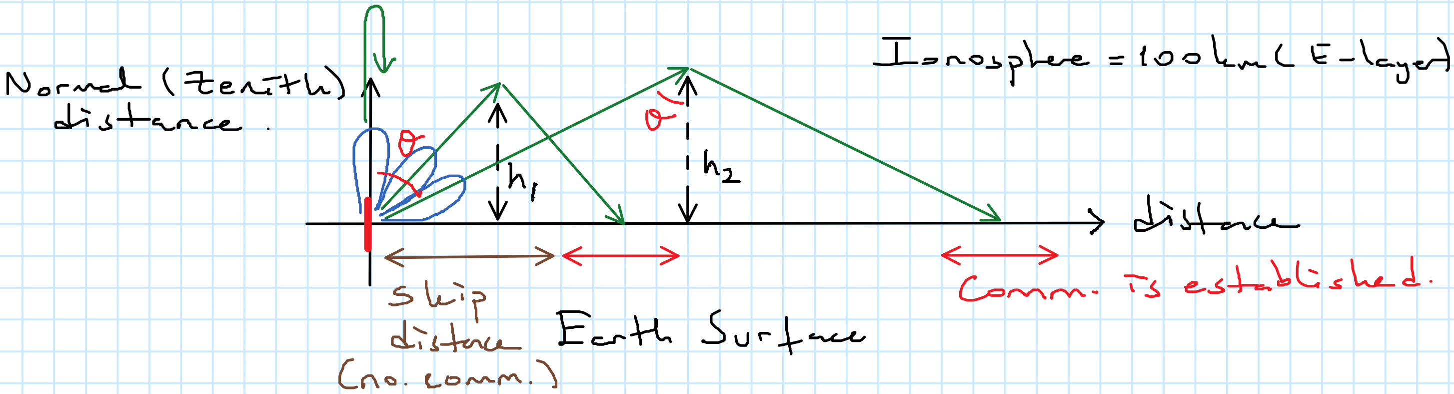
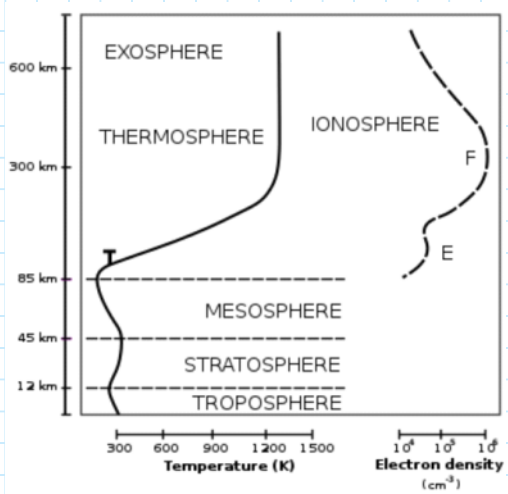
Monday, May 31, 2021 2:33 PM

2-) Sky-Wave Propagation.

As waves travel through the ionosphere, they may reflect from a height h, and come back to the earth's surface at a distance d from the transmitter





Maximum Usable Frequency (MUF):

Any E.M wave whose frequency below MUF reflects from the ionosphere from a height h. Waves with frequency above MUF escape to space.

- There is also the lowest usable frequency (LUF), and optimum working frequency (OWF).

By definition:

$$MUF = \frac{CF}{\cos\theta}$$

where CF = Critical frequency : Max freq for reflection at zenith.

θ = Angle of incidence.

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It is given that

$$OWF = (0.85)\,MUF$$ (Best freq for skywave comm.)

Also,

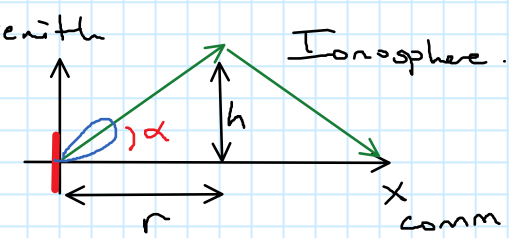
$$CF = 9\sqrt{N_{max}}$$

where

N_{max} = Maximum e^- density

Generally,

- Frequencies below 10MHz propagate most efficiently by skywaves at night
- Frequencies above 10MHz propagate most efficiently during the day
- At night, skip distance is increased.
- In VHF (30-300MHz), the skywave comm is not possible.
- Usually, HF (High Frequency → 3-30MHz) is used for sky wave comm.
- In attempt to calculate the radiation distance of the lowest angle beam (take-off angle), we can use the equation.



$$\tan\alpha = \frac{h}{r}$$

or

$$r = \frac{h}{\tan\alpha}$$

and the comm. distance $= 2r = \dfrac{2h}{\tan\alpha}$, where $\alpha = 90° - \theta$

—o—

P48

Monday, May 31, 2021 3:25 PM

Ex:

Given that the maximum e^- density in the ionosphere is $N = 5\times10^4 \frac{e^-}{cm^3}$ and $\theta = 70°$. Find the MUF and OWF.

Ans.

$$N_{max} = 5\times10^4 e^- . cm^{-3} = 5\times10^{10} e^- . m^{-3}$$

$$\Rightarrow CF = 9\sqrt{N_{max}} = 9\sqrt{5\times10^{10}} = 2\times10^6 \ Hz = 2\ MHz.$$

$$\Rightarrow MUF = \frac{CF}{\cos\theta} = \frac{2\times10^6}{\cos 70°} = 5.884\ MHz \approx 6\ MHz.$$

$$OWF = 0.85\ MUF = 5\ MHz.$$

- As a rule of thumb, MUF is about 3 times of CF.

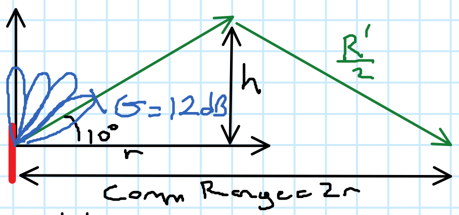
At $\theta = 30°$

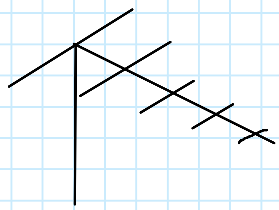
$$MUF = \frac{2\times10^6}{\cos 30°} = 2.3\ MHz.$$

- MUF $\propto$ θ (Angle of incidence)

Ex.

For a horizontal polarized log-periodic dipole antenna with 1kW of radiated power, has the following radiation pattern.





Evaluate the maximum comm range through sky waves at a single hop of the smallest angle beam, and evaluate the power density at this distance. (h = 100km → E-layer)

Ans.

$$Range = \frac{2h}{\tan\alpha} = \frac{2(10^5)}{\tan 10°} = \frac{200000}{0.17} = 1176.47\ km.$$

$$P = P_{iso} . Gain = P_i . G = \frac{P_{rad}}{4\pi R^2} . G = \frac{1000}{4\pi R^2}(10^{1.2}) = \frac{1000\ (10^{1.2})}{4\pi(1194000)^2}$$

↳ range of the wave propagation $= R'$

$$\cos\alpha = \frac{r}{\frac{R'}{2}} \Rightarrow R' = \frac{1176.4km}{\cos 10°}$$

$$= \frac{1176.4\ km}{\cos 10°} = 1194\ km \quad \Rightarrow P = 8.85\times10^{-10} = 885\ \frac{pW}{m^2}$$

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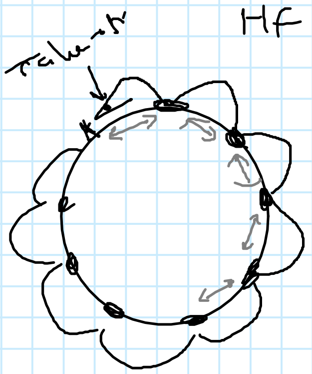


HF – Long wave.

– Ducting: Changes in sky-wave comm. due to seasonal and atmospheric variations

3-) Ground Wave Propagation:

The electric field expression for a ground wave:

$$E = E_0 e^{-\alpha z} e^{-j\beta z} \left(\frac{V}{m}\right)$$

or $E(t,z) = E_0 e^{-\alpha z} \cos(\omega t - \beta z) \left(\frac{V}{m}\right)$ $\qquad c = \frac{1}{\sqrt{\mu\epsilon}}$

where α = Attenuation constant

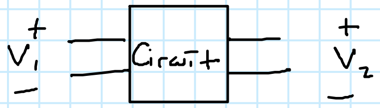
β = Phase constant. $\left(\beta = \omega\sqrt{\mu\epsilon} = 2\pi f\sqrt{\mu\epsilon} = \frac{2\pi f}{c} = \frac{2\pi}{\lambda}\right)$ $\left(\frac{1}{m}\right)$

Unit of α = ?

The unit of α is Nepers

Neper:

By definition, the voltage gain of the following circuit is



$$G_v = \frac{V_2}{V_1}$$

or in nepers:

$$G_{v_{Np}} = \ln\frac{V_2}{V_1} = \ln V_2 - \ln V_1$$

Conversion from dB to Neper:

$$G_{dB} = 10\log_{10}\frac{P_2}{P_1} = 10\log_{10}\left(\frac{V_2}{V_1}\right)^2 = 20\log_{10}\left(\frac{V_2}{V_1}\right)$$

Also,

$$G_{Np} = \ln\frac{V_2}{V_1} = \frac{\log_{10}\frac{V_2}{V_1}}{\log_{10} e} = \frac{\frac{G_{dB}}{20}}{\log_{10} e}$$

$$\Rightarrow \frac{G_{dB}}{G_{Np}} = 20\log_{10} e \quad \text{where } e = 2.71828\ldots$$

$$= 8.868585 \ \frac{dB}{Np}.$$

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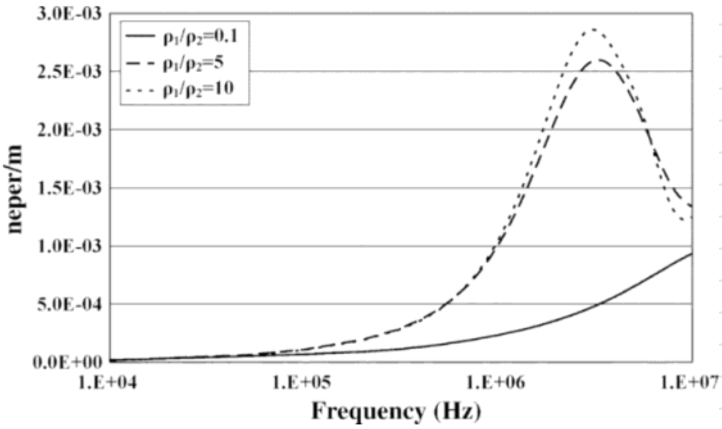
or

$$\frac{G_{Np}}{G_{dB}} = 0.11513 \ \frac{Np}{dB}$$



TY - JOUR
AU - Papadopoulos, Theofilos
AU - Papagiannis, Grigoris
AU - Labridis, Dimitris
PY - 2009/03/01
SP - 1064
EP - 1067
T1 - Wave Propagation Characteristics of Overhead Conductors Above Imperfect Stratified Earth for a Wide Frequency Range
VL - 45
DO - 10.1109/TMAG.2009.2012580
JO - Magnetics, IEEE Transactions on ER

Neper Analysis

$$G_{V_{Np}} = \ln \frac{V_2}{V_1} \ (Np)$$

Let us call $\frac{V_2}{V_1} = m$.

Then,

$$G_V = \ln m$$

or

$$m = e^{G_V}$$ → in Np.

Since for a ground wave,

$$E = E_0 e^{-\alpha z} e^{-j\beta z} \left(\frac{V}{m}\right)$$

Attenuation term

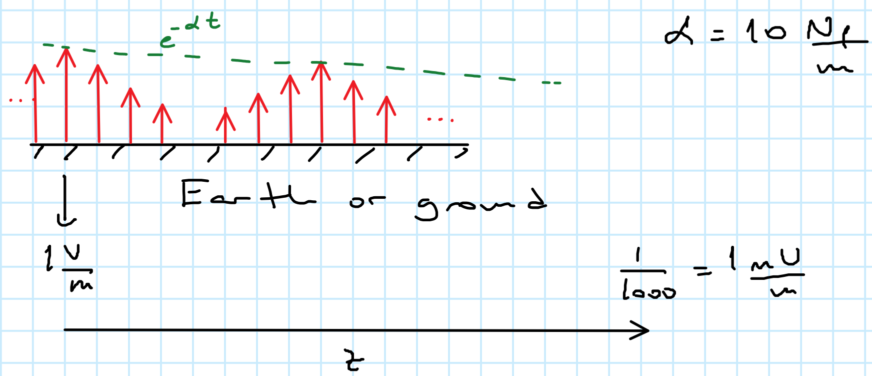
where $\alpha z = Np$

$$\Rightarrow \boxed{\alpha = \frac{Np}{m}}$$ (attenuation constant)

Ex:

A vertical polarized dipole antenna radiates ground waves. The attenuation is 10^{-3} Np/m. Find the distance at which the amplitude drops to 1000 of its original value?

Ans:



$\alpha = 10 \ \frac{Np}{m}$

$$E_0 e^{-\alpha z} = \frac{E_0}{1000}$$

$$e^{-10^{-3} z} = \frac{1}{1000}$$

$$e^{10^{-3} z} = 1000 \Rightarrow 10^{-3} z = \ln 1000 \Rightarrow z = 6.9 \ km.$$

Sample Exam Questions:

1 Question 1: (30 points)

An electromagnetic wave is transmitted at 30MHz. Find the wave velocity, wavelength, attenuation constant and phase constant in
a) air, given that $\epsilon_r = 1$, $\mu_r = 1$, $\sigma = 0$ (10 points).
b) sea water, given that $\epsilon_r = 70$, $\mu_r = 1$, and $\sigma = 5\ S/m$ (10 points).
c) concrete, given that $\epsilon_r = 20$, $\mu_r = 1$, and $\sigma = 0.01\ S/m$ (10 points).
($\epsilon_0 = 8.854 \times 10^{-12}$ and $\mu_0 = 4\pi \times 10^{-7}$)

2 Question 2 (25 points)

A uniform plane wave is normally incident from air to flat dielectric with $\epsilon_r = 16$ (Gallium arsenide). Given $|E_i| = E_0 = 10mV/m$, find

a-) The reflection and transmission coefficients ? (12 points)
b-) The average power densities for all waves, confirm the conservation of energy ? (13 points)

3 Question 3 (20 points)

A transmitter antenna radiates electromagnetic waves at 200W of total output power. Evaluate the power density of the waves at R=1km away from this antenna in the broadside direction where the antenna has 10dB gain.

4 Question 4 (25 points)

A log periodic antenna has the following radiation pattern

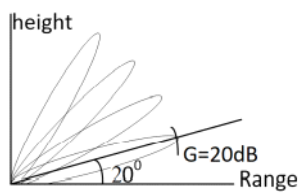


Figure 1: Radiation pattern.

The frequency of operation is 6 MHz. The antenna is aimed to operate with sky wave communication at lowest angle beam (20^0) for maximum range. The E-layer reflection height is 100km. Given $N = 9 \times 10^{10}\ (\frac{e^-}{m^3})$,
a-) Find the MUF and OWF ? (9 points)
b-) Does this wave reflect, why ? If it reflects, find the max. comm. range ? (8 points)
c-) Find the power density of the wave at the max. comm. range if $P_t = 1kW$. (8 points)

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Sample Exam Solutions:

	Exact	Good dielectric $\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \ll 1$	Good conductor $\left(\frac{\sigma}{\omega\varepsilon}\right)^2 \gg 1$
Attenuation constant α	$= \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}-1\right]\right\}^{1/2}$	$\simeq \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
Phase constant β	$= \omega\sqrt{\mu\varepsilon}\left\{\frac{1}{2}\left[\sqrt{1+\left(\frac{\sigma}{\omega\varepsilon}\right)^2}+1\right]\right\}^{1/2}$	$\simeq \omega\sqrt{\mu\varepsilon}$	$\simeq \sqrt{\frac{\omega\mu\sigma}{2}}$
Wave Z_w intrinsic η_c impedances $Z_w = \eta_c$	$= \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}$	$\simeq \sqrt{\frac{\mu}{\varepsilon}}$	$\simeq \sqrt{\frac{\omega\mu}{2\sigma}}(1+j)$
Wavelength λ	$= \frac{2\pi}{\beta}$	$\simeq \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$	$\simeq 2\pi\sqrt{\frac{2}{\omega\mu\sigma}}$
Velocity v	$= \frac{\omega}{\beta}$	$\simeq \frac{1}{\sqrt{\mu\varepsilon}}$	$\simeq \sqrt{\frac{2\omega}{\mu\sigma}}$
Skin depth δ	$= \frac{1}{\alpha}$	$\simeq \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}$	$\simeq \sqrt{\frac{2}{\omega\mu\sigma}}$

1-) $f = 30\,MHz$.

a-) in air:

$\frac{\sigma}{\omega\epsilon} = 0$ Good dielectric.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{30\times10^6} = \frac{30}{3} = 10\,m.$$

$$v = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3\times10^8\ m/s$$

$$\alpha = 0,\quad \beta = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0} = \frac{2\pi(3\times10^7)}{3\times10^8} = \frac{2\pi}{10} = \frac{\pi}{5} \cong 0.63$$

b-) in sea water:

$$\frac{\sigma}{\omega\epsilon} = \frac{5}{(2\pi\times30\times10^6)(70)(8.854\times10^{-12})} = 43 \Rightarrow \text{Good conductor.}$$

$$v = \sqrt{\frac{2\omega}{\mu\sigma}} = \sqrt{\frac{2\cdot2\pi(30\times10^6)}{(4\pi\times10^{-7})(5)}} = 7.74\times10^6\ m/s$$

$$\lambda = 2\pi\sqrt{\frac{2}{\omega\mu\sigma}} = 2\pi\sqrt{\frac{2}{2\pi(30\times10^6)(4\pi\times10^{-7})5}} = 2\pi\sqrt{\frac{1}{4\pi^2\times5}} = \frac{2\pi}{2\pi}\sqrt{\frac{1}{5}} = \sqrt{\frac{1}{5}}$$

$$= 0.4472\,m = 44.7\,cm.$$

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = \left[\frac{2\pi(30\times10^6)(4\pi\times10^{-7})5}{2}\right]^{\frac{1}{2}} = \left[4\pi^2(3\times10^7)(10^{-7})5\right]^{\frac{1}{2}}$$

$$= 2\pi\sqrt{15} = 24.33\ Np/m.$$

$$\beta = \alpha = 24.33$$

c-) in concrete: $\frac{\sigma}{\omega\epsilon} = \frac{0.01}{(2\pi\times30\times10^6)(20)(8.854\times10^{-12})} = 0.3 \Rightarrow \left(\frac{\sigma}{\omega\epsilon}\right)^2 = 0.05 \ll 1.$

Good dielec.

$$v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\mu_0\epsilon_r\epsilon_0}} = \frac{1}{\sqrt{20}\sqrt{\mu_0\epsilon_0}} = \frac{3\times10^8}{\sqrt{20}} = 6.7\times10^7\ m/s.$$

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{2\pi}{2\pi\times30\times10^6\sqrt{20}\sqrt{\mu_0\epsilon_0}} = \frac{3\times10^8}{30\times\sqrt{20}\times10^6} = \frac{10}{\sqrt{20}} = 2.236\,m.$$

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}} = \frac{0.01}{2}\sqrt{\frac{\mu_0}{\epsilon_0}\cdot\frac{1}{20}} = \frac{0.01}{2}\cdot377\cdot\frac{1}{\sqrt{20}} = 0.42\ Np/m.$$

$$\beta = \omega\sqrt{\mu\epsilon} = \frac{2\pi\times30\times10^6\times\sqrt{20}}{3\times10^8} = \frac{2\pi}{10}\times\sqrt{20} = 2.8$$

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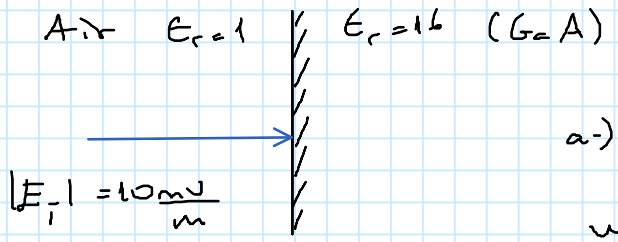
2-)



a-) $\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1}$ and $T = \frac{2\eta_2}{\eta_1+\eta_2}$

where

$\eta_1 = 377,\ \eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}} = \frac{377}{\sqrt{16}} = \frac{377}{4} = 94.25\Omega.$

$\Rightarrow \Gamma = \frac{94.25-377}{94.25+377} = -0.6$

$T = \frac{2(94.25)}{94.25+377} = 0.4.$

b-) $\bar{P}_{i_{avg}} = \hat{a}_z \frac{E_o^2}{2\eta_1} \left(\frac{W}{m^2}\right) = \hat{a}_z \frac{(0.01)^2}{2\cdot 377} = 1.326\times10^{-7}\,W$

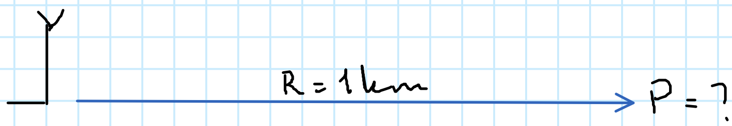
$\bar{P}_{r_{avg}} = -\hat{a}_z |\Gamma|^2 |P_{i_{avg}}| \left(\frac{W}{m^2}\right) = -\hat{a}_z (0.6)^2 \cdot (1.326\times10^{-7}) = 0.477\times10^{-7}$

$\bar{P}_{t_{avg}} = \hat{a}_z |T|^2 \frac{\eta_1}{\eta_2} \cdot |\bar{P}_{i_{avg}}| = \hat{a}_z [1-|\Gamma|^2] |\bar{P}_{i_{avg}}|$

$= \hat{a}_z [1-(0.6)^2] (1.326\times10^{-7})$

$= \hat{a}_z\, 0.849\times10^{-7}$

3-)



$P_{rad} = 200\,W$

$G = 10$

$P = P_{iso}\, G = \frac{200}{4\pi R^2}\cdot 10 = \frac{2000}{4\pi(1000)^2} = 1.6\times10^{-4}$

$= 0.16\,\frac{mW}{m^2}.$

4-) $f = 6\,MHz.$ $h = 100\,km.$, $N = 9\times10^{10}\ e^-.cm^{-3}$

a-) $CF = 9\sqrt{N_{max}} = 9\sqrt{9\times10^{10}} = 27\sqrt{10^{10}} = 27\times10^5 = 2.7\,MHz$

$MUF = \frac{CF}{\cos\theta} = \frac{2.7MHz}{\cos 70^\circ} = 7.89\,MHz. \approx 7.9\,MHz.$

$OWF = 0.85 \times MUF = 0.85\times 7.9MHz \approx 6.72\,MHz.$

b-) Yes, it reflects because $f = 6\,MHz < MUF = 7.9\,MHz$

$Range = \frac{2h}{\tan\alpha} = \frac{2\times100\,000}{\tan 20^\circ} = 5.49\times10^5 = 549\,km. \approx 550\,km.$

c-) $R' = \frac{550\,km}{\cos 20^\circ} = 585\,km.$ $\Rightarrow P = \frac{1000}{4\pi(585\,000)^2}\cdot 10^2 = 2.32\times10^{-8}\,\frac{W}{m^2}$

$= 23.2\,\frac{nW}{m^2}.$

↓ FINAL EXAM STARTS FROM HERE! ↓

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Effects of E.M. Radiation on Human Health:

According to the Standardization Agreement (Stanag) evolution and control of Personal Exposure to Radio Frequency Fields between 3 kHz to 300 GHz:

Frequency Range (MHz)	Electric Field (V/m)	Magnetic Field (A/m)	Power Density (W/m^2)
0.003-0.1	614	163	10^3
0.1-3	614	16.3/f	10^3
3-30	1842/f	16.3/f	$900/f^2$
30-100	61.4	16.3/f	10
100-300	61.4	16.3/f	10
300-3000			f/30

f: Frequency in MHz

Maximum values for safety.

According to Consumer and Clinical Radiation Protection Bureau, Health Canada:

Kaynak <https://www.canada.ca/en/health-canada/services/publications/health-risks-safety/limits-human-exposure-radiofrequency-electromagnetic-energy-range-3-300.html#s2.2.2>

Frequency (MHz)	Electric Field Strength (E_{RL}), (V/m, RMS)	Magnetic Field Strength (H_{RL}), (A/m, RMS)	Power Density (S_{RL}), (W/m^2)	Reference Period (minutes)
10 - 20	27.46	0.0728	2	6
20 - 48	$58.07 / f^{0.25}$	$0.1540 / f^{0.25}$	$8.944 / f^{0.5}$	6
48 - 300	22.06	0.05852	1.291	6
300 - 6000	$3.142 f^{0.3417}$	$0.008335 f^{0.3417}$	$0.02619 f^{0.6834}$	6
6000 -15000	61.4	0.163	10	6
15000 - 150000	61.4	0.163	10	$616000 / f^{1.2}$
150000 - 300000	$0.158 f^{0.5}$	$4.21 \times 10^{-4} f^{0.5}$	$6.67 \times 10^{-5} f$	$616000 / f^{1.2}$

Frequency, *f*, is in MHz.

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Other standards are shown below. (AN OVERVIEW OF STANDARDS AND REGULATION CONCERNING EXPOSURE TO RADIOFREQUENCY FIELDS Annamaria PALJANOS, paljanosanna@yahoo.com Călin MUNTEANU calin.munteanu@et.utcluj.ro TECHNICAL UNIVERSITY CLUJ NAPOCA, ROMANIA)

Frequency range (MHz)		3-10	10-100	100-300
Electric field strength (V/m)	IEEE – 2345 Zone 0	823.8/f	27.5	27.5
	IEEE – 2345 Zone 1	1842/f	61.4	61.4
	IEEE – General public	823.8/f	27.5	27.5
	IEEE Controlled environment	1842/f	61.4	61.4
	ICNIRP General public	87/[redacted]	28	28
	ICNIRP Occupational exposure	610/f	61	61
	EU Recommendation	87/[redacted]	28	28
	EU Directive	610	61	61
Magnetic field strength (A/m)	IEEE – 2345 Zone 0	16.3/f	158.3/[redacted]	0.0729
	IEEE – 2345 Zone 1	16.3/f	16.3/f	0.163
	IEEE – General public	16.3/f	158.3/[redacted]	0.0729
	IEEE Controlled environment	16.3/f	16.3/f	0.163
	ICNIRP General public	0.73/f	0.073	0.073
	ICNIRP Occupational exposure	1.6/f	0.16	0.16
	EU Recommendation	0.73/f	0.073	0.073
Power density (W/[redacted])	IEEE – 2345 Zone 0	([redacted])	(2, 94 x [redacted])	2
	IEEE – 2345 Zone 1	(90[redacted])	([redacted])	10
	IEEE – General public	([redacted])	(2, 94 x [redacted])	2
	IEEE Controlled environment	(90[redacted])	([redacted])	10
	ICNIRP General public	-	2	2
	ICNIRP Occupational exposure	-	10	10
	EU Recommendation	-	2	2
	EU Directive	-	50	50

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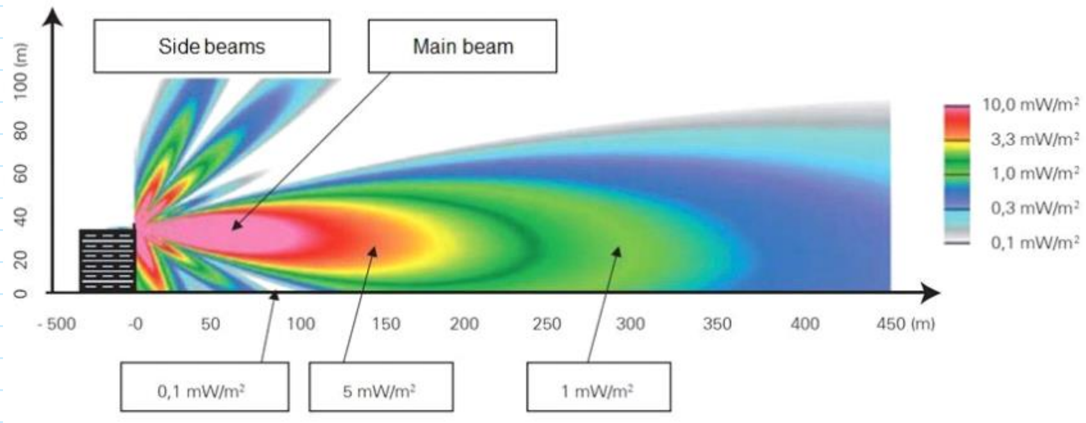
Ex

Mobile phone base station antenna radiates waves according to the figure below.



Evaluate the range from transmitter at which it is considered to be safe for human health. $P_{total} = 50W$, $f = 1GHz$.

Ans:

From the figure,

at $R = 150m$, $P = 5\frac{mW}{m^2}$

We have

$$P = P_{iso} \cdot G = \frac{50}{4\pi(150)^2} \cdot G = 5\times10^{-3}$$

$$\Rightarrow G = \frac{5\times10^{-3}\times4\pi\times(150)^2}{50} = 28.3$$

$$G_{dB} = 10\log_{10} 28.3 = 14.5 \text{ dB}.$$

To evaluate the safe distance (according to stanag)

$$P_{safe} = \frac{f}{30} = \frac{1000}{30} = P_{iso} \cdot G(\theta = \tfrac{\pi}{2}) = \frac{50}{4\pi R^2} \cdot (28.3)$$

$$\Rightarrow R_{safe} = \sqrt{\frac{30 \cdot 50 \quad (28.3)}{1000 \quad 4\pi}} = 1.84 m. \approx 2m.$$

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To evaluate the safe distance: (according to Canada Heath Bureau)

$$P_{safe} = 0.02619 f^{0.6834} = 3 = P_{iso} \cdot G(\theta = \frac{\pi}{2}) = \frac{50}{4\pi R^2} \cdot (28.3)$$

$$\Rightarrow R_{safe} = \sqrt{\frac{50 \; (28.3)}{3 \cdot 4\pi}} = 1.84 \text{ m.} \approx 6.1 \text{ m.}$$ (exposure time = 6 min.)

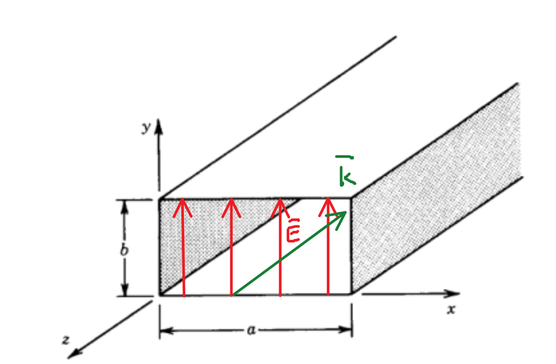
According to EU recommendation (assuming P=2 W/m² at 1GHz)

$$P_{safe} = 2 = P_{iso} \cdot G = \frac{P_t}{4\pi R^2} \cdot G = \frac{50}{4\pi R^2} (28.3)$$

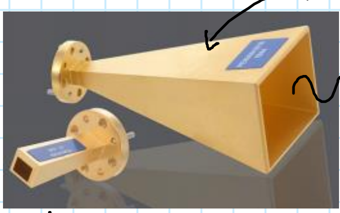
$$\Rightarrow R \approx 7.5 \text{ m}$$

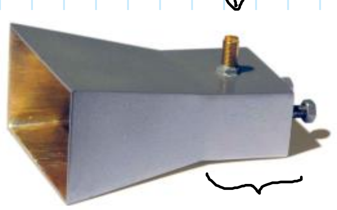
- Rectangular WaveGuides and Cavity Resonators -

- Waveguides are conducting structures in which the electromagnetic waves propagate.



- Waveguides are usually used when very high power waves feed a transmitter antenna.





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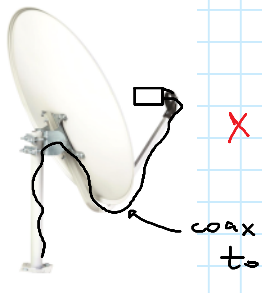
- Waveguides are resistive against damages from heat because of their structure.



T_x antenna

T_x antenna

- Only TM or TE waves propagate inside waveguides. TEM mode is not supported inside waveguides.

TE^z mode analysis:

- The wavenumber

$$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z$$

and

$$|k| = [k_x^2 + k_y^2 + k_z^2]^{\frac{1}{2}}$$

or

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

where

k_z = Wavenumber along the z-direction. (z is the direction of energy propagation)

Thus,

$$k^2 - k_z^2 = k_x^2 + k_y^2$$

When $k_z = 0$ (cut-off) $\Rightarrow$ No propagation along the z-direction.

$$\Rightarrow k_c^2 = k_x^2 + k_y^2 \quad \text{(cut-off)}$$

where

$$k_x^2 = \left(\frac{m\pi}{a}\right)^2 \quad \text{and} \quad k_y^2 = \left(\frac{n\pi}{b}\right)^2$$

where $m = 0, 1, 2, \ldots$ and $n = 0, 1, 2, \ldots$

and $m \neq n$.

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Also, k_c = cut-off wavenumber $= \omega_c\sqrt{\mu\epsilon}$ inside air

where ω_c = cut-off frequency in rad/sec.

For $\omega > \omega_c \rightarrow$ Propagation

$\omega = \omega_c \rightarrow$ No propagation

$\omega < \omega_c \rightarrow$ Attenuation

Dominant mode = The mode of waves inside a waveguide with the smallest ω_c for m and n

This is TE modes with m=1, n=0

Let us find the dominant mode expression.

$$k_c^2 = k_x^2 + k_y^2$$

or

$$\left(\omega_c\sqrt{\mu\epsilon}\right)^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$\Rightarrow \omega_c^2 = \frac{1}{\mu\epsilon}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]$$

or

$$\omega_c = \frac{1}{\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}}$$

or

$$f_{c_{mn}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{\frac{1}{2}} (Hz)$$

(cut-off frequency)

- Thus, waveguides behave like high pass filters, because for $f \leq f_c$ there is no propagation, for $f > f_c$ there is propagation
- For air waveguide $\frac{1}{\sqrt{\mu\epsilon}} = c = 3\times 10^8$ m/s.

$$\Rightarrow f_{c_{mn}} = \frac{c}{2\pi}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right]^{1/2} (Hz)$$

- For $a > b$, TE_{10} is the dominant mode.
- For $b > a$, TE_{01} is the dominant mode

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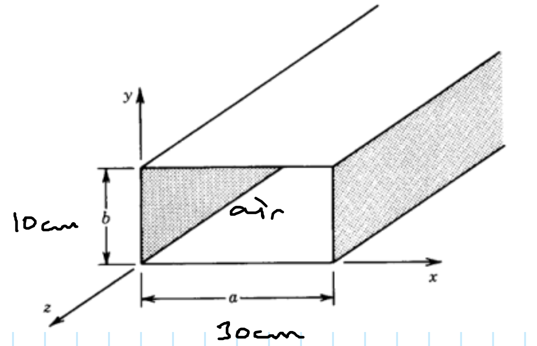
Ex:

A rectangular waveguide of dimensions $a = 30\,cm$, $b = 10\,cm$ as shown below.



a-) Find the dominant mode, the dominant mode frequency and first 3 modes and their cut-off frequencies for TE waves.

b-) For a signal at $f = 1.7\,GHz$, which modes propagate inside this waveguide?

<u>Ans</u>

a-) For air waveguide with $a > b$, the dominant mode is TE_{10} where the cut-off frequency is

$$f_c = \frac{c}{2\pi}\left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right] \ (Hz)$$

Thus,

$$f_{c_{10}} = \frac{3\times10^8}{2\pi}\left[\left(\frac{\pi}{a}\right)^2\right]^{1/2} = \frac{3\times10^8}{2\pi}\cdot\frac{\pi}{0.3}$$

$$= \frac{3\times10^8}{2\times\frac{3}{10}} = \frac{10^9}{2} = 0.5\times10^9$$

$$\Rightarrow f_{c_{10}} = 500\times10^6 = \boxed{500\ MHz}$$

The other 3 cut-off frequencies are

$$f_{c_{01}} = \frac{3\times10^8}{2\pi}\left(\frac{\pi}{b}\right) = \frac{3\times10^8}{2\times0.1} = 1.5\times10^9 = \boxed{1.5\ GHz}$$

$$f_{c_{11}} = \frac{3\times10^8}{2\pi}\underbrace{\left[\left(\frac{\pi}{0.3}\right)^2 + \left(\frac{\pi}{0.1}\right)^2\right]^{\frac{1}{2}}}_{\approx 33} = \frac{3\times10^8}{2\pi}\cdot 33 = 15.76\times10^8 = \boxed{1.576\ GHz}$$

$f_{c_{21}} \overset{?}{<} f_{c_{12}}$ ✓ or $f_{c_{21}} \overset{?}{>} f_{c_{12}}$ = ?

since $a > b$

$$f_{c_{21}} = \frac{3\times10^8}{2\pi}\left[\left(\frac{2\pi}{0.3}\right)^2 + \left(\frac{\pi}{0.1}\right)^2\right]^{\frac{1}{2}} = \boxed{1.8\ GHz}$$

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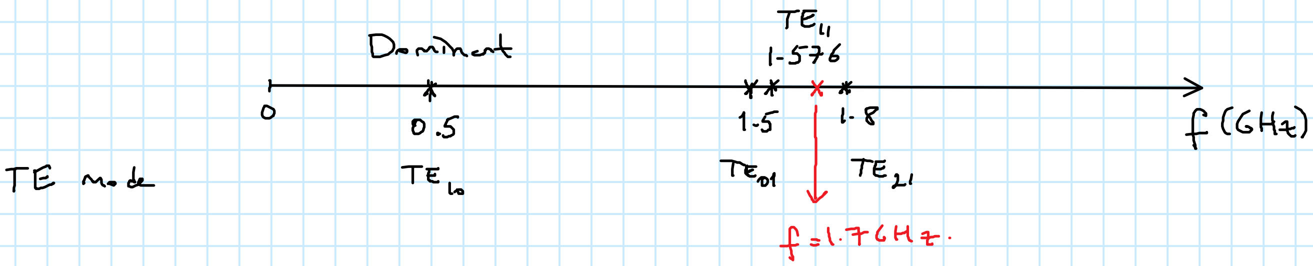
Wednesday, August 11, 2021 12:48 PM

b-) Thus, on the frequency axis, let us show these cut-off frequencies:

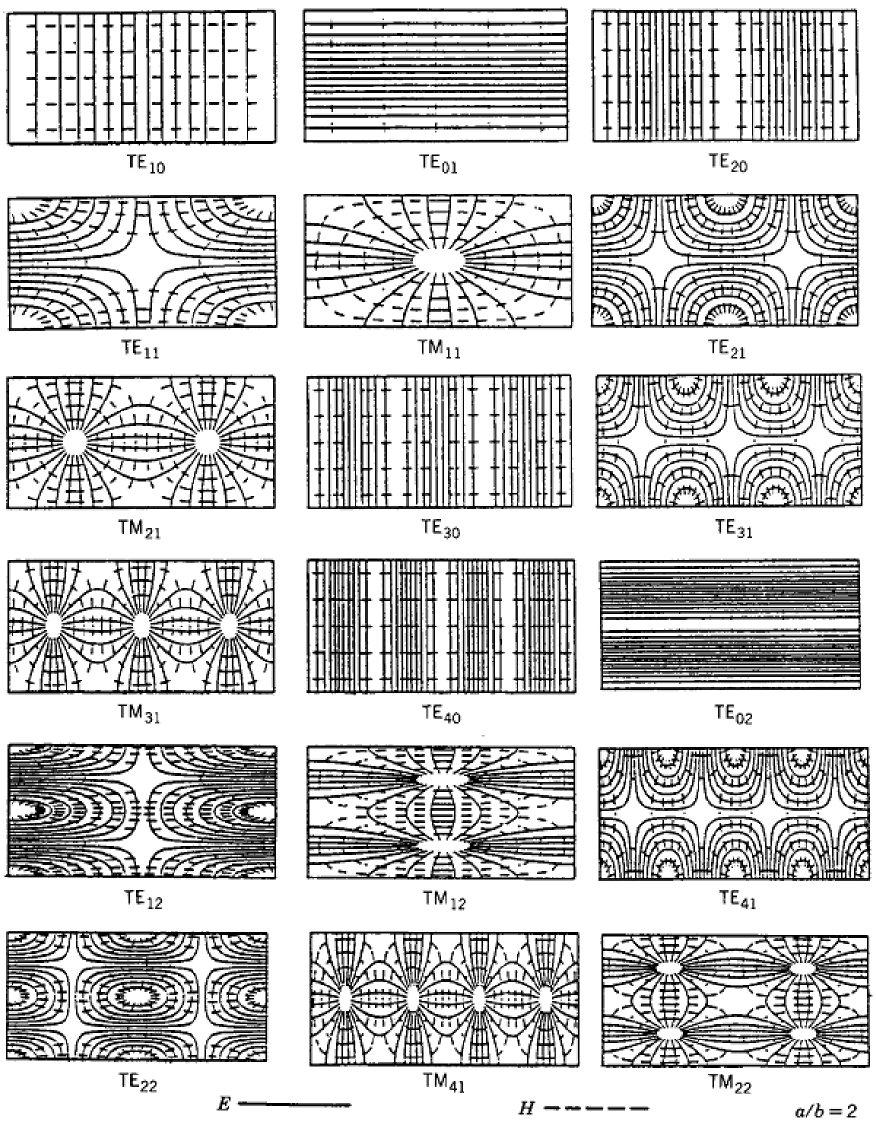


TE_{10} propagates ✓

TE_{01} " ✓

TE_{11} " ✓

—o—



(Field patterns for various modes)

P62

Thursday, August 12, 2021 11:30 AM

2-) TM^z mode analysis.

– The wavenumber

$$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z$$

and

$$|k| = \left[k_x^2 + k_y^2 + k_z^2\right]^{\frac{1}{2}}$$

or

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

where

k_z = Wavenumber along the z-direction. (z is the direction of energy propagation)

Thus,

$$k^2 - k_z^2 = k_x^2 + k_y^2$$

When $k_z = 0$ (cut-off) ⇒ No propagation along the z-direction.

$$\Rightarrow k_c^2 = k_x^2 + k_y^2 \quad \text{(cut-off)}$$

where

$$k_x^2 = \left(\frac{m\pi}{a}\right)^2 \quad \text{and} \quad k_y^2 = \left(\frac{n\pi}{b}\right)^2$$

where $m = 1, 2, ..$ and $n = 1, 2, ...$

Also,

$$f_{c_{mn}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}, \quad m = 1, 2, .. \quad n = 1, 2, ..$$

The dominant mode:

$$(f_c)_{11} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{1}{2a\sqrt{\mu\epsilon}}\sqrt{1 + \left(\frac{a}{b}\right)^2}$$

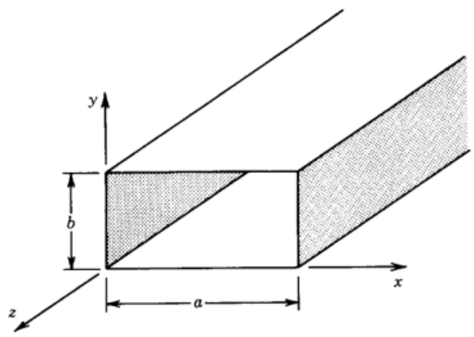
P63

Thursday, August 12, 2021 11:42 AM

Ex

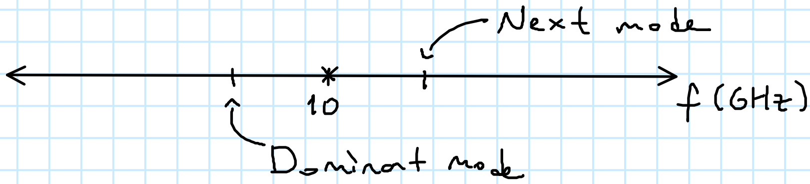
Design a rectangular waveguide with dimensions a and b (a>b) that will operate at a single mode at 10 GHz. Assume an air filled waveguide.

Ans:



a = ?

b = ?



Let us choose the dominant mode to be 9 GHz.

$$\Rightarrow \quad \frac{c}{2}\sqrt{\frac{1}{a^2}+\frac{1}{b^2}} = 9\times 10^9$$

Find $\frac{1}{a^2}+\frac{1}{b^2}$

$$\Rightarrow \frac{1}{a^2}+\frac{1}{b^2} = \left(\frac{2\times 9\times 10^9}{3\times 10^8}\right)^2 = (60)^2$$

Let $a = 5\,cm = 0.05\,m$

$$\Rightarrow \frac{1}{(0.05)^2} + \frac{1}{b^2} = (60)^2 = 3600$$

$$\frac{1}{b^2} = 3600 - \frac{1}{(0.05)^2}$$

$$\frac{1}{b^2} = 3200$$

$$b = \sqrt{\frac{1}{3200}}$$

$$\Rightarrow b = 0.01767\,m$$

$$= 1.767\,cm.$$

Note: $\frac{a}{b} < 3$ is necessary for realistic implementation.

In our case, $\frac{a}{b} = \frac{5}{1.767} = 2.83 < 3$, it is realistic.

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Checking the next mode : TM^{12} or TM^{21} ✓ since $a > b$

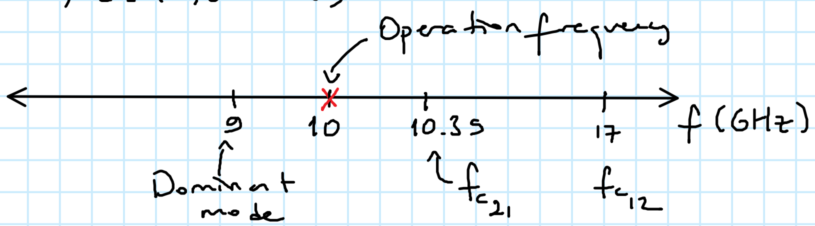
$$f_{c_{21}} = \frac{1}{2\pi\sqrt{\mu\epsilon}}\sqrt{\left(\frac{2\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$

$$= \frac{c}{2\pi}\sqrt{\left(\frac{2\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2}$$

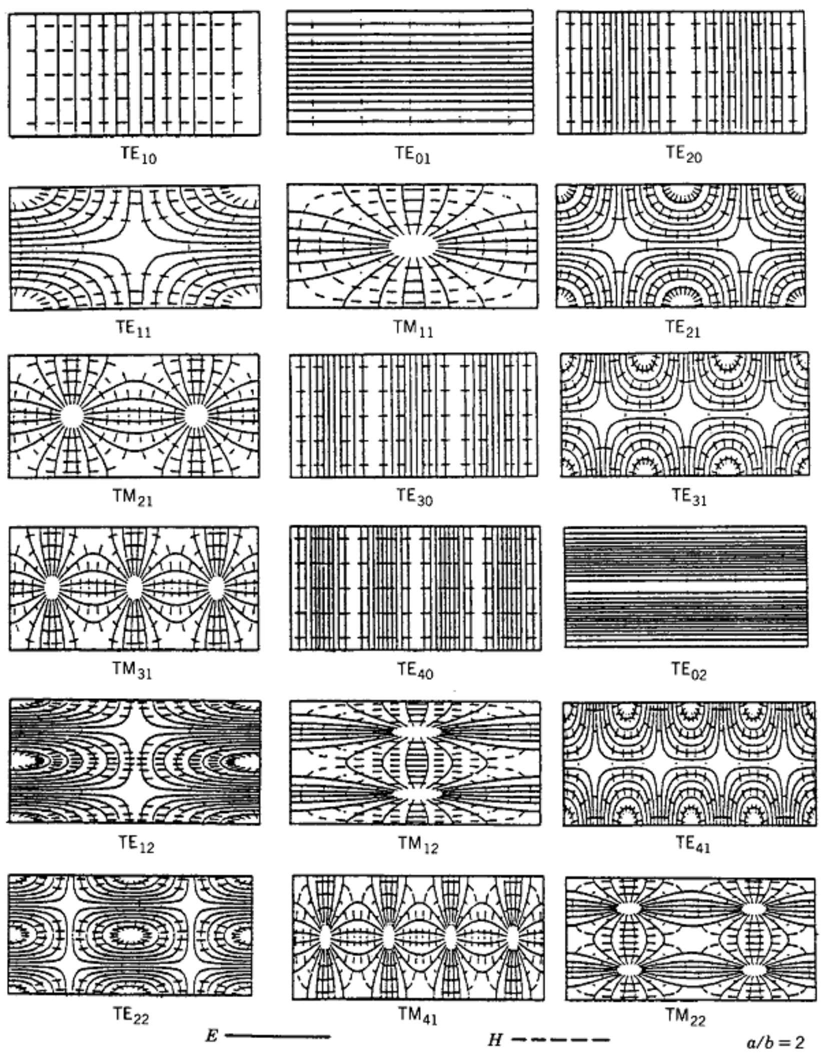
$$= \frac{3\times 10^8}{2\pi}\sqrt{\left(\frac{2\pi}{0.05}\right)^2 + \left(\frac{\pi}{0.01767}\right)^2}$$

$$= 10.39 \text{ GHz} \quad , \left(f_{c_{12}} = 17 \text{ GHz.}\right)$$

Thus, for $a = 5$ cm, $b = 1.767$ cm,



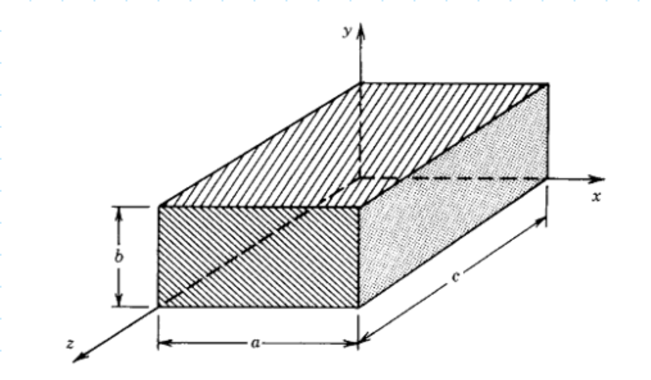
—o—



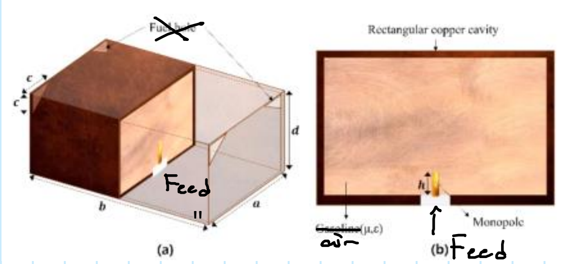
(Field patterns for various modes)

Rectangular Cavity Resonators

- Resonator means oscillator. Thus, cavity resonators are microwave devices that generate certain frequency waves. The frequency is determined by the dimensions of the resonator.
- Rectangular cavity resonators are 3D rectangular boxes with hollow inside, usually air filled.



- It supports TE and TM modes only.
- The energy is coupled to the device from probes or holes on one of its sides.



1-) TE^z mode.

- The wavenumber

$$\bar{k} = \hat{a}_x k_x + \hat{a}_y k_y + \hat{a}_z k_z$$

and

$$|k| = [k_x^2 + k_y^2 + k_z^2]^{\frac{1}{2}}$$

or

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

where

k_z = Wavenumber along the z-direction. (z is the direction of energy propagation)

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Thus,

$$k^2 = k_x^2 + k_y^2 + k_z^2$$

When $k = 0$ (resonance) → oscillation.

$$\Rightarrow k_r^2 = k_x^2 + k_y^2 + k_z^2 \quad \text{(resonance)}$$

where

$$k_x^2 = \left(\frac{m\pi}{a}\right)^2, \quad m = 0, 1, 2, \ldots$$

$$k_y^2 = \left(\frac{n\pi}{b}\right)^2, \quad n = 0, 1, 2, \ldots \qquad m \neq n \neq 0$$

$$k_z^2 = \left(\frac{p\pi}{c}\right)^2, \quad p = 1, 2, 3, \ldots$$

Thus,

$$f_{r_{mnp}}^{TE} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

for $m = 0, 1, 2, \ldots$, $n = 0, 1, 2, \ldots$, $p = 1, 2, 3, \ldots$ and $m \neq n \neq 0$.

The dominant mode is

$$f_{r_{101}}^{TE} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{c}\right)^2} \quad (Hz)$$

2-) TM^z mode:

Similar analysis are carried out with the following

$$f_{r_{mnp}}^{TM} = \frac{1}{2\pi\sqrt{\mu\epsilon}} \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{c}\right)^2}$$

where $m = 1, 2, 3, \ldots$, $n = 1, 2, 3, \ldots$, $p = 0, 1, 2, \ldots$

The dominant mode is TM_{110}

$$f_{r_{110}}^{TM} = \frac{1}{2\sqrt{\mu\epsilon}} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{c}{2} \sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \quad (Hz)$$

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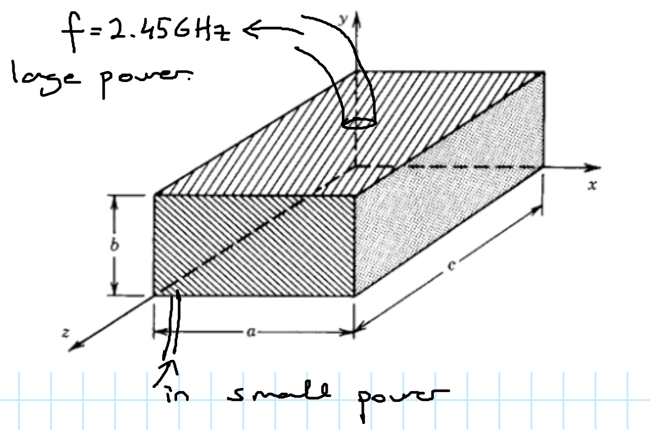
Ex:

Design a rectangular cavity that generates 2.45 GHz E.M. waves. (TM mode)

Ans:



$a = ?$

$b = ?$

$c = ?$

For TM dominant mode:

speed of light.

$$f_{r_{110}}^{TM} = \frac{1}{2\sqrt{\mu\epsilon}}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} = \frac{c}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2} \quad (Hz)$$

$$2.45\times10^{9} = \frac{3\times10^{8}}{2}\sqrt{\left(\frac{1}{a}\right)^2 + \left(\frac{1}{b}\right)^2}$$

Let $a = 10\,cm = 0.1\,m$.

$$\Rightarrow \quad 2.45\times10^{1} = \frac{3\times10^{0}}{2}\sqrt{\left(\frac{1}{0.1}\right)^2 + \left(\frac{1}{b}\right)^2}$$

$\vdots$

$\Rightarrow b = 0.07743\,m = 7.743\,cm$

Let $c = 0.2\,m$.

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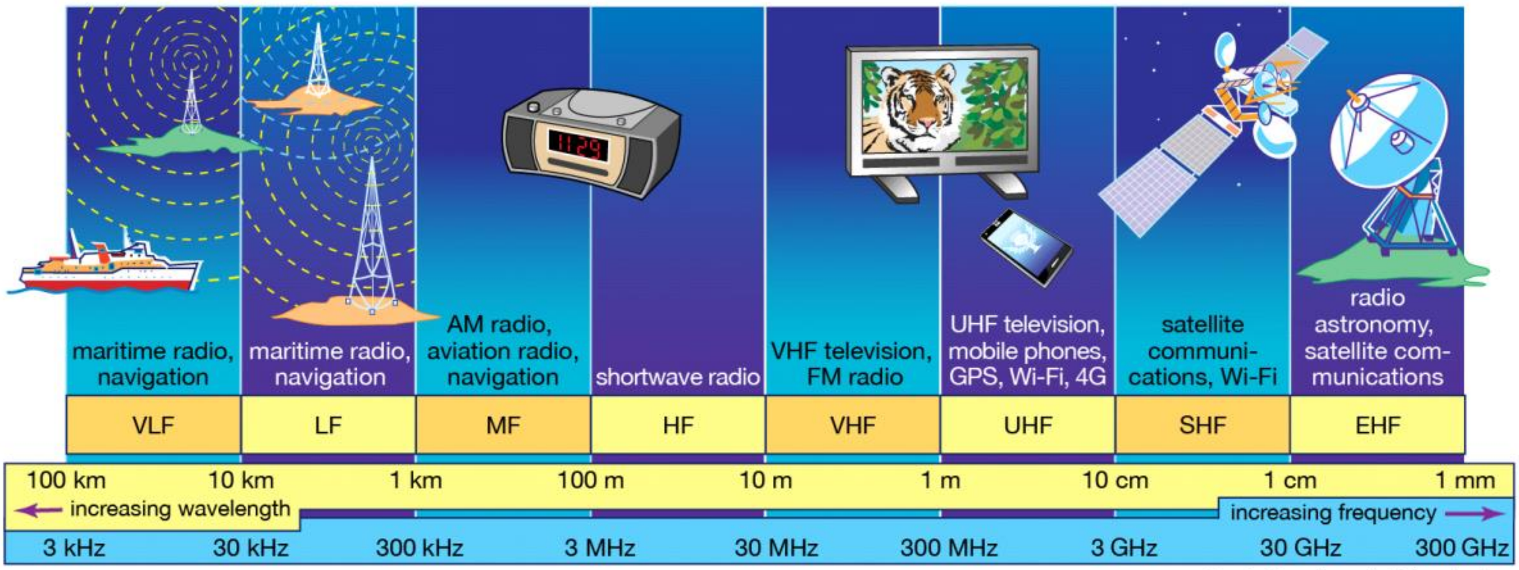
- Frequency Spectrum -

Extremely Low Freq (ELF): 0-3 kHz
Very Low Frequency (VLF): 3-30 kHz.
Low Freq (LF): 30-300 kHz
Medium Freq (MF) 300 kHz - 3 MHz
High Freq (HF) 3 MHz - 30 MHz
Very High Freq (VHF) 30 MHz - 300 MHz
Ultra High Freq (UHF) 300 MHz - 3 GHz
Super High Freq (SHF) 3 GHz - 30 GHz
Extremely High Freq (EHF) 30 GHz - 300 GHz
Optical Freq. 300 GHz → 3 THz.

8 GHz - 12 GHz: X-band.



- EE311 Final Exam Date: 31-08-2021 Tuesday at 13:00.
- The exam will be online with the same zoom id and psswrd in the lectures.
- The exam will be open-notes.
- The exam topics start from the midterm until the end of the lectures.
- Use of phones and computers are not allowed during the exam.

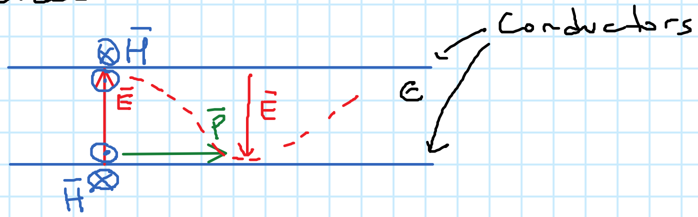
P69

Friday, August 13, 2021 3:24 PM

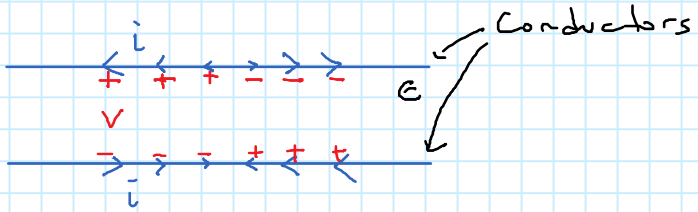
– E.M. Waves Inside Infinite Transmission Lines –

– Transmission line is a structure that transmits e.m. waves from a location to another location in terms of TEM waves.

– Transmission line is made of two conductors separated by a dielectric material

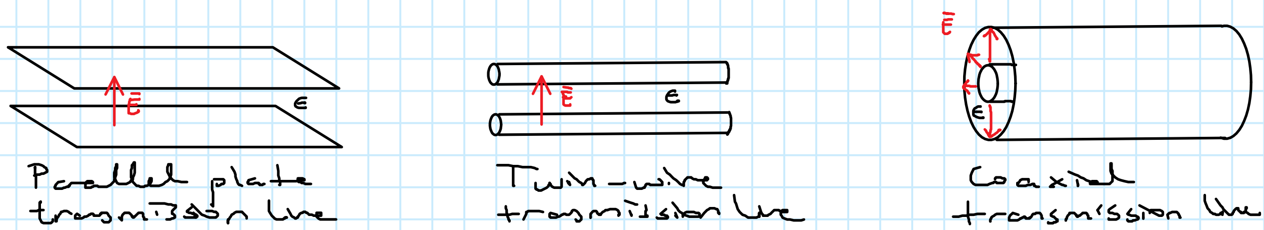


– Voltage and current are created along the conductors such as

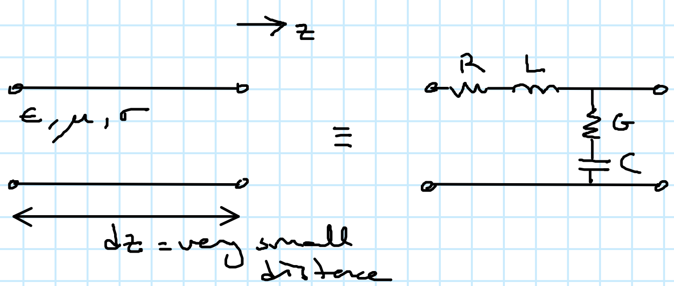


– Thus, the voltage V and current i inside the transmission line behaves as if they propagate. This is why they are called propagating "voltage wave" and propagating "current wave".

– Some of the transmission line structures are:



A part of a transmission line can be modelled as the following circuit:



P70

Wednesday, August 18, 2021 12:00 PM

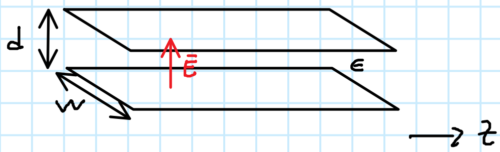
- There are 2 types of transmission lines
  1-) Lossy transmission lines 2-) Lossless transmission lines
- Usually, if a line is relatively short, it can be taken as lossless
- For lossless lines, $R = G = 0$, and there is only L and C.

Lossless Lines:

- Many times, line resistance and conductance can be taken as zero, especially when it is not a long line
- For lossless transmission lines:

$$L = \mu \frac{d}{w} \quad (H/m)$$

is the inductance per unit length of the parallel transmission line. Here, d and w are the thickness and width of the transmission line respectively.



The capacitance per unit length is given as

$$C = \epsilon \frac{w}{d} \quad \left(\frac{F}{m}\right)$$

The voltage wave and current wave are expressed as

$V^+(z) = V_o e^{-j\beta z}$ (incident voltage wave phasor expression)

$V^-(z) = V_o e^{j\beta z}$ (reflected " " " ")

$i^+(z) = I_o e^{-j\beta z}$ (incident current " " ")

$i^-(z) = I_o e^{j\beta z}$ (reflected " " " ").

Total expressions

$$V(z) = V^+(z) + V^-(z) = V_o e^{-j\beta z} + V_o e^{j\beta z}$$

$$i(z) = i^+(z) + i^-(z) = I_o e^{-j\beta z} + I_o e^{j\beta z}$$

At any point on the line,

$$Z_o = \frac{V^+(z)}{I^+(z)} = \frac{V^-(z)}{I^-(z)} = \sqrt{\frac{L}{C}} \quad (\Omega)$$

is the "characteristic impedance" of the line.

Also,

$$\beta = \omega\sqrt{LC} = \omega\sqrt{\mu\epsilon}$$

is the "phase constant."

The phase velocity of the waves is

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu\epsilon}} \quad (m/s)$$

Lossy Parallel Plate Transmission Lines:

It is given that

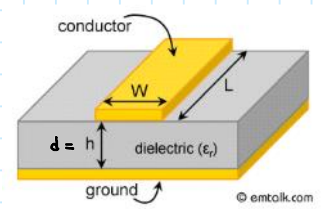
$$G = \sigma\frac{w}{d} \quad \left(\frac{S}{m}\right)$$

and

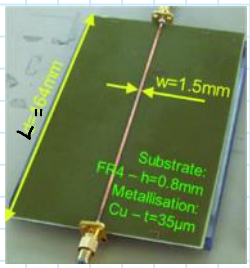
$$R = \frac{2}{w}\sqrt{\frac{\pi f \mu}{\sigma}} \quad \left(\frac{\Omega}{m}\right)$$

where f is the Hz frequency

- An example to parallel plate transmission lines is "microstrip lines".



Schematic view



Real view

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Ex:

Neglecting losses and assuming the substrate of a stripline to have a thickness 0.4 mm and a dielectric constant 2.25,

a-) Determine the required width w of the line to have $Z_0 = 50\,\Omega$.

b-) Determine L, C of the line.

c-) Determine v_p along the line

d-) Repeat a, b and c for $Z_0 = 75\,\Omega$.

Ans:

a-) $Z_0 = \sqrt{\frac{L}{C}} = \frac{d}{w}\sqrt{\frac{\mu}{\epsilon}} = \frac{d}{w}\eta$

$$\Rightarrow w = \frac{d}{Z_0}\underbrace{\sqrt{\frac{\mu}{\epsilon}}}_{\eta} = \frac{0.4\times10^{-3}}{50}\cdot\underbrace{\frac{\eta_0}{\sqrt{\epsilon_r}}}_{\eta} = \frac{0.4\times10^{-3}\times377}{50\sqrt{2.25}} = 2\times10^{-3}\,m = 2\,mm$$

b-) $L = \mu\frac{d}{w} = (4\pi 10^{-7}) \times \frac{0.4}{2} = 2.51\times10^{-7}\ \left(\frac{H}{m}\right)$ or $0.251\left(\frac{\mu H}{m}\right)$. ($\rightarrow \mu_0$)

$C = \epsilon_0\epsilon_r\frac{w}{d} = \left(\frac{10^{-9}}{36\pi}\right)\times 2.25\times\frac{2}{0.4} = 99.5\times10^{-12}\left(\frac{F}{m}\right)$ or $99.5\left(\frac{pF}{m}\right)$. ($\rightarrow \epsilon_0 = 8.854\times10^{-12}$)

c-) $v_p = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}} = \frac{c}{\sqrt{2.25}} = \frac{c}{1.5} = 2\times10^{8}\ \left(\frac{m}{s}\right)$

d-) Since w is inversely proportional to Z_0, we have for $Z_0' = 75\,\Omega$

$$w' = \left(\frac{Z_0}{Z_0'}\right)w = \frac{50}{75}\times 2 = 1.33\ mm.$$

$$L' = \left(\frac{w}{w'}\right)L = \left(\frac{2}{1.33}\right)\times 0.251 = 0.377\ \left(\frac{\mu H}{m}\right)$$

$$C' = \left(\frac{w'}{w}\right)C = \left(\frac{1.33}{2}\right)\times 99.5 = 66.2\ \left(\frac{pF}{m}\right)$$

$$v_p' = v_p = 2\times10^{8}\left(\frac{m}{s}\right).$$

For general transmission lines (lossy lines)

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}}\ (\Omega)$$

P73

Wednesday, August 18, 2021 12:57 PM

The voltage and current waves are:

$$V^+(z) = V_0 e^{-j\gamma z} \quad , \quad V^-(z) = V_0 e^{j\gamma z}$$

where γ = Propagation constant.

$$\gamma = \alpha + j\beta$$

where α = Attenuation constant ($\frac{Np}{m}$)

β = Phase constant ($\frac{rad}{m}$)

and

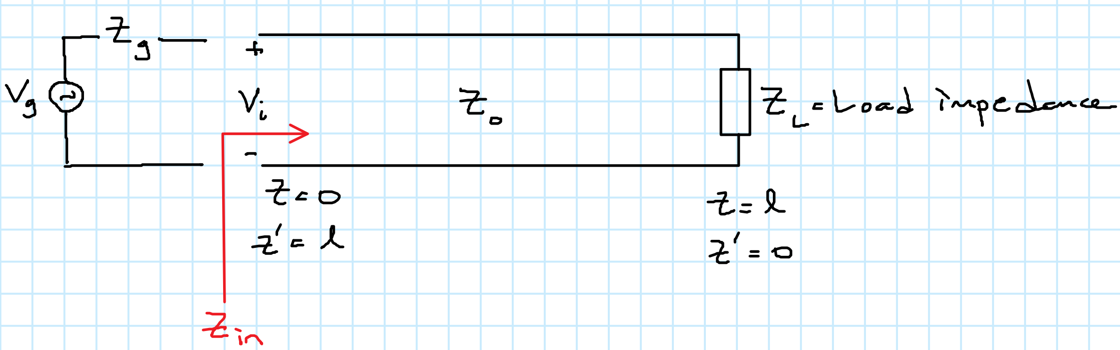
$$V(z) = V^+(z) + V^-(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z}$$

$$i(z) = i^+(z) + i^-(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$Z_0 = \frac{V^+(z)}{i^+(z)} = \frac{V^-(z)}{i^-(z)} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

– Loaded Transmission Lines –

- The transmission line is ended with a load:



$$Z_L = \frac{V_L}{i_L}$$

$$Z_0 = \frac{V^+(z)}{i^+(z)} \quad , \quad Z_{in} \neq Z_0 \quad , \quad Z_{in} = \frac{V(z)}{i(z)} = \frac{V^+(z) + V^-(z)}{i^+(z) + i^-(z)}$$

For lossless lines:

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tan\beta l}{Z_0 + Z_L \tan\beta l}}$$

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Wednesday, August 18, 2021 1:38 PM

Ex!

A signal generator with voltage

$$V_g(t) = 0.3\cos(2\pi 10^8 t)\ \text{V}$$

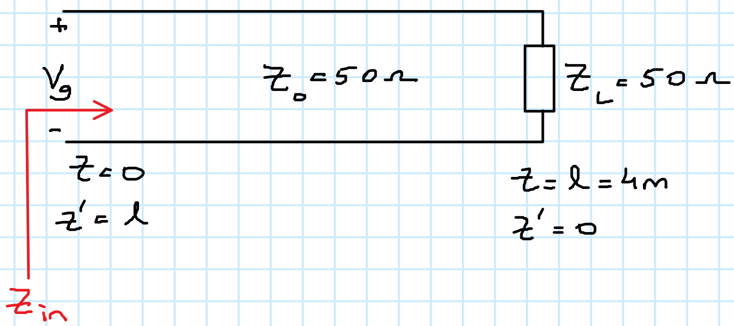
is connected to $50\,\Omega$ lossless transmission line. The line is 4m long, and $u_p = 2.5\times10^8$ m/s. For a match load,

a-) Instantenous expressions of the voltage and current waves.

b-) Instantenous expressions of the voltage and current at the load

c-) Average power transmitted to the load

Ans:



The given quantities are:

$\tilde{V}_g = 0.3\angle 0°$ (phasor with cos reference)

$Z_0 = 50$

$\omega = 2\pi\times10^8$ rad/s.

$u_p = 2.5\times10^8$ m/s

$l = 4$ m.

We have a match load means $V^-(z) = I^-(z) = 0$.

a-) $\tilde{V}_i(z) = V_0^+ e^{-j\beta z} = 0.3\,e^{-j\beta z}$

where $\beta = \dfrac{\omega}{u_p} = \dfrac{2\pi\times10^8}{2.5\times10^8} = 0.8\pi\ (\text{rad/m})$

$\Rightarrow \tilde{V}(z) = \tilde{V}^+(z) = 0.3\,e^{-j0.8\pi z}$ V,

and $\tilde{I}(z) = \tilde{I}^+(z) = \dfrac{V^+(z)}{Z_0} = \dfrac{0.3}{50}\,e^{-j0.8\pi z}$ A.

b-) For $z = l = 4m$, $\quad Re[0.3\, e^{-j0.8.\pi.4}\, e^{j\omega t}]$

$\bar{V}_L = V(4) = V^+(4) = 0.3\, e^{-j0.8.\pi.4}$ (phasor) $\Rightarrow \quad V_L(t) = 0.3 \cos(2\times10^8 t - 10)$ V

$\bar{i}_L = i(4) = I^+(4) = \frac{V^+(4)}{Z_0} = \frac{0.3}{50}\, e^{-j0.8\pi.4} \Rightarrow i_L(t) = 6\cos(2\times10^8 t - 10)$ mA

c-) $P_{avg} = \frac{1}{2} Re[\bar{V}_L . \bar{i}_L^*]$

$= \frac{1}{2} Re[0.3\, e^{-j0.8\pi.4} . 0.006\, e^{j0.8\pi 4}] = \frac{1}{2}(0.3 \times 0.006)$

$= 0.9$ mW.

- When a transmission line is terminated in a load impedance Z_L different from the characteristic impedance Z_0, both an incident wave (from the generator) and a reflected wave (from the load) exist.

$$V_i(z) = V^+(z) = V_0^+ e^{-j\beta z}$$

$$V_r(z) = V^-(z) = V_0^- e^{j\beta z} = V_0^+ . \Gamma . e^{j\beta z}$$

where Γ = Reflection coefficient $= |\Gamma| . e^{j\theta_r}$

$$|\Gamma| = \frac{V_0^-}{V_0^+} \quad \text{and} \quad \boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Standing wave ratio:

$$\boxed{S = \frac{|V_{max}|}{|V_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|}}$$

For $\Gamma = 0$, $S = 1$, $Z_L = Z_0$ (Matched load)

$\Gamma = -1$, $S = \infty$, $Z_L = 0$ (Short circuit)

$\Gamma = 1$, $S = \infty$, $Z_L = \infty$ (Open circuit)

Ex:

A 50Ω trasmission line is terminated with a load $Z_L = 200\Omega$.

a-) Find the reflection coefficient and VSWR $(S) = ?$

b-) Assume air line with frequency $f = 1\,GHz$, find the expression for $V^+(z)$, $V^-(z) = ?$ Given $V_0^+ = 1\,V$.

Ans:

a-) $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200 - 50}{200 + 50} = \frac{\cancel{150}^{3}}{\cancel{250}_{5}} = \frac{3}{5} = 0.6$ (60% is reflected)

$VSWR = S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.6}{1 - 0.6} = \frac{1.6}{0.4} = \frac{16}{4} = 4$

$|\Gamma|_{dB} = 20 \log_{10}^{0.6} = -4.43\,dB \quad (S_{11})$

} Not accepted
VSWR < 2 is accepted.
$|\Gamma|_{dB} < -10\,dB$ " .

b-) ϵ_0, μ_0 , $\omega = 2\pi (10^9)$ rad/sec.

$$\beta = \frac{\omega}{c} = \frac{2\pi \times 10^9}{3 \times 10^8} = \frac{20\cancel{\pi}}{\cancel{3}} = 20 \text{ rad/m}$$

$$V^+(z) = V_0^+ e^{-j20z} = e^{-j20z}$$

$$V^-(z) = V_0^+ |\Gamma| e^{-j20z} = 0.6\, e^{-j20z}$$

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- Midterm Exam Solutions -

1-) $f = 3\,MHz$:

a-) In air: $\sigma = 0 \Rightarrow$ Good dielectric

$$\beta = \omega\sqrt{\mu_0\epsilon_0} = 2\pi \times 3\times10^6 \times \frac{1}{c} = \frac{2\pi\times3\times10^6}{3\times10^8} = 2\pi\times10^{-2} = 6.28\times10^{-2} = 0.0628$$

$$v_p = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = c = 3\times10^8\ m/s.$$

$$\alpha = 0\ ,\quad \lambda = \frac{c}{f} = \frac{3\times10^8}{3\times10^6} = 100\ m.$$

b-) Sea water: $\epsilon_r = 70$, $\mu_r = 1$, $\sigma = 5\ S/m$

$$\frac{\sigma}{\omega\epsilon} = \frac{5}{(2\pi\times3\times10^6)(70\times8.854\times10^{-12})} = 428 \gg 1 \Rightarrow \text{Good conductor!}$$

$$\Rightarrow \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi\times3\times10^6\times4\pi\times10^{-7}\times5}{2}} = 7.7$$

$$v_p = \sqrt{\frac{2\omega}{\mu\sigma}} = \sqrt{\frac{2\times2\pi\times3\times10^6}{4\pi\times10^{-7}\cdot5}} = 2.45\times10^6\ m/s$$

$$\alpha = \beta = 7.7\ ,$$

$$\lambda = 2\pi\sqrt{\frac{2}{\omega\mu\sigma}} = 2\pi\sqrt{\frac{2}{2\pi\times3\times10^6\times4\pi\times10^{-7}\times5}} = 0.8165\ m = 81.65\ cm$$

2-) $\eta_1 = 377\,\Omega$, $\eta_2 = \frac{1}{\sqrt{\epsilon_r}}\eta_1 = \frac{1}{2}\eta_1$

a-)
$$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{1}{2}\eta_1-\eta_1}{\frac{1}{2}\eta_1+\eta_1} = \frac{\frac{1}{2}-1}{\frac{1}{2}+1} = \frac{-\frac{1}{2}}{\frac{3}{2}} = -\frac{1}{2}\cdot\frac{2}{3} = -\frac{1}{3} = -0.3\bar{3}$$

$$T = 1-|\Gamma| = 1-0.\bar{3} = 0.\bar{6}$$

b-)
$$|P^i_{avg}| = \frac{E_0^2}{2\eta_1} = \frac{(0.1)^2}{2(377)} = 1.327\times10^{-5} = 13.27\ \mu W/m^2$$

$$|P^r_{avg}| = |\Gamma|^2\,|P^i_{avg}| = (-0.\bar{3})^2\,(1.327\times10^{-5}) = 1.47\ \mu W/m^2$$

$$|P^t_{avg}| = (1-|\Gamma|^2)|P^i_{avg}| = |T|^2\frac{\eta_1}{\eta_2}|P^i_{avg}| = [1-(0.\bar{3})^2]\cdot(13.27\ \mu W/m^2) = 11.8\ \mu W/m^2$$

$$\Rightarrow |P^i_{avg}| = |P^r_{avg}| + |P^t_{avg}|\ \checkmark$$

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3-) To find A :

$$\bar{A} = \frac{\mu}{4\pi} \int_C \bar{I} \cdot \frac{e^{-jkR}}{R} dl'$$

or

$$\bar{A} = \frac{\mu_0}{4\pi} \int_C (\hat{a}_z I_0) \frac{e^{-jkr}}{r} dz'$$

$$\Rightarrow \bar{A} = \frac{4\pi \times 10^{-7}}{4\pi} \int_{-D/2}^{D/2} \hat{a}_z I_0 z^4 \frac{e^{-jkr}}{r} dz'$$

$$\int_{-\frac{D}{2}}^{\frac{D}{2}} z^4 dz = \frac{1}{5} z^5 \Big|_{-\frac{D}{2}}^{D/2} = \frac{1}{80} D^5$$

$$\Rightarrow \bar{A} = \hat{a}_z 10^{-7} \cdot I_0 \cdot \frac{e^{-jkr}}{r} \frac{D^3}{12}$$

$$\Rightarrow \bar{E} = -j\omega \bar{A}_\theta \Rightarrow \bar{E}_\theta = -j\omega 10^{-7} \cdot I_0 \frac{e^{-jkr}}{r} \cdot \frac{D^5}{80} \sin\theta \left(\frac{V}{m}\right)$$

or

$$\bar{E}_\theta = 10^{-7} \omega I_0 \frac{D^5}{80} \sin\theta \frac{e^{-jkr}}{r} e^{-j\frac{\pi}{2}}$$

$$\bar{E}_\theta(r,t) = Re\left[\bar{E}_\theta \cdot e^{j\omega t}\right] = \underbrace{10^{-7} \omega I_0 \frac{D^5}{80} \frac{1}{r} \cdot \overbrace{\sin\theta}^{f(\theta) = \text{Pattern function}}}_{E_0} \cdot \cos(\omega t - \underbrace{kr}_{\text{phase factor}} - \frac{\pi}{2})$$

4-) $P_t = 1kW$

$R = 10km$

$G = 10$

$$P = \frac{P_t}{4\pi R^2} \cdot G = \frac{1000}{4\pi (10000)^2} \cdot 10 = 8 \mu W/m^2$$

—o—

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